Chapter 10: The Manipulability of Voting Systems
Lesson Plan

- An Introduction to Manipulability
- Majority Rule and Condorcet's Method
- The Manipulability of Other Voting Systems for Three or More Candidates
- Impossibility
- The Chair's Paradox

Chapter 10: The Manipulability of Voting Systems
An Introduction to Manipulation in Voting

- Manipulation in Voting
  - In the process of voting, you misrepresent your actual preferences on your ballot hoping to strategically achieve the election result that you prefer.
  - Voting manipulation is often referred to as **strategic voting**.

- Insincere Ballot or Disingenuous Ballot
  - An insincere ballot is the term given to a ballot that misrepresents a voter's true preference.

Chapter 10: The Manipulability of Voting Systems

An Introduction to Manipulation in Voting

- Definition of Manipulability
  - A voting system is said to be manipulable if there exist two sequences of preference-list ballots and a voter (call the voter Jane) such that:
    1. Neither election results in a tie.
    2. The only ballot change is by Jane.
    3. Jane prefers (assuming that her ballot in the first election represents her true preferences) the outcome of the second election to that of the first election.

  - A voting system is manipulable if there is at least one scenario in which some voter can achieve a more preferred election outcome by unilaterally changing his/her ballot.
    - Unilateral change in a ballot is when there is only one voter changing his/her ballot.

Chapter 10: The Manipulability of Voting Systems
Majority Rule and Condorcet's Method

- May's Theorem for Manipulability
  - Among all two-candidate voting systems that never result in a tie, majority rule is the only one that treats all voters and both candidates equally and is monotone and nonmanipulable.
  - Monotone means that a single voter's change in ballot from a vote for the loser to a vote for the winner has no effect on the election outcome.
  - For the two-candidate case, monotonicity and nonmanipulability mean the exact the same thing.

Thus, a majority-rule voting system cannot be manipulated.

Since Condorcet's method is based on majority rule and we know majority rule is nonmanipulable, then it follows that **Condorcet's method is nonmanipulable** also.

Majority Rule and Condorcet's Method

- The Nonmanipulabilty of Condorcet's Method
  - Condorcet's method is nonmanipulable in the sense that a voter can never unilaterally change an election result from one candidate to another candidate that he or she prefers.

Majority Rule and Condorcet's Method

- Example: Election with Three Candidates—A, B, C
  - Using Condorcet's method, suppose you, as one of the voters, prefer candidate A to candidate B, but B wins the election.
  - With B winning, Condorcet's method means that B defeated every other candidate in a one-on-one contest, based on the ballots cast.
  - Even with your original ballot with A over B, candidate B still won more than half of the other voters that ranked B over A for this contest.
  - If you change your ballot to prefer B over A, then B would still defeat A—and this is not what your original true preference was.
  - Note: Using Condorcet's method, you could change the election by changing your ballot to result in a tie for some other candidate; but you can never change the election to have your true preference (your first choice of all the candidates) win if that candidate lost.

Other Voting Systems for Three or More Candidates

- Nonmanipulability of the Borda Count—Three Candidates
  - The Borda count cannot be manipulated with three candidates. A voter cannot unilaterally change an election outcome from one single winner to another single winner that he/she truly preferred in the first election.
- Manipulability of the Borda Count—Four or More Candidates
  - The Borda count can be manipulated with four or more candidates and two or more voters. There exists an election in which a voter can unilaterally change the election outcome from one single winner to another single winner that he or she truly preferred originally in the first election.

Election 1: Voters' true preferences

Borda scores: A=4, B=5, C=3, D=0

B wins.

Election 1			
Rank	**Voters (2)**		Points
First	A	B	3
Second	B	C	2
Third	C	A	1
Fourth	D	D	0

Election 2: Voter 1 wants A to win and he manipulates list.

New Borda scores: A=4, B=3, C=3, D=2

A wins.

Election 2			
Rank	**Voters (2)**		Points
First	A	B	3
Second	**D**	C	2
Third	**C**	A	1
Fourth	**B**	D	0

Other Voting Systems for Three or More Candidates

Agenda Manipulation of Sequential Pairwise Voting

- Suppose we have four candidates and three voters who we know will be submitting the following preference lists ballots.
- Now suppose we have agenda-setting power and we get to choose the order in which the one-on-one contests will take place.
- With the power to set the agenda, we can arrange for whichever candidate we want the winner to be.

Agenda Manipulation – Those in control of procedures can manipulate the agenda by restricting alternatives [candidates] or by arranging the order in which they are brought up.

For example:
Candidates that appear later in the agenda are favored over those that appear early in the agenda.

Rank	Voters (3)		
First	A	C	B
Second	B	A	D
Third	D	B	C
Fourth	C	D	A

Agenda B, C, D, A: A wins.

Other Voting Systems for Three or More Candidates

- The Group Manipulability of Plurality Voting
  - Plurality voting cannot be manipulated by a single individual.
  - However, it is group manipulable, meaning that there are elections in which a group of voters can change their ballots so that the new winner is preferred to the old winner by everyone in the group, assuming that the original ballots represent the true preferences of each voter in the group.
    - Since only first-place votes are considered, a voter cannot manipulate the vote to get his/her true preference to win.
    - However, insincere voting can take place when someone does not want to vote for his/her true preference because it may in a sense be "throwing away your vote," as some accuse Nader voters in Florida of doing in the 2000 election.
- Voting Systems for Three or More Candidates
  - Other voting systems that are manipulable by a single voter, even in the case of three voters and three candidates, are the plurality run-off rule, the Hare system, and sequential pairwise voting.

Impossibility

- <u>Four Desirable Properties of Condorcet's Method</u>:
  - Elections never result in ties.
  - It satisfies the Pareto condition.
  - It is nonmanipulable.
  - It is not a dictatorship.

  However, for Condorcet's voting paradox, we know there are elections that produce no winner at all.

<u>Question</u>: Is there a voting system that satisfies all four of these properties and (unlike Condorcet's method) always yields a winner?

- Even if we try to loosen the rules (examples: break ties with a fixed ordering list, let the Cordorcet's winner be the candidate with the best "win-loss record" in one-on-one contests, or Copeland's rule), the answer is still ***NO.***
- <u>Gibbard-Satterthwaite Theorem</u> – With three or more candidates and any number of voters, there does not exist—and there never will exist—a voting system that always produces a winner, never has ties, satisfies the Pareto condition, is nonmanipulable, and is not a dictatorship.

The Chair's Paradox

The Chair's Paradox

- The chair paradox can occur if the voter with tie-breaking power (the "chair") ends up with his/her least-preferred candidate as the election winner.
- Example:

  Suppose we have three candidates—A, B, and C—and three voters whom we'll call the chair, you, and me. The chair will have tie-breaking power—if each candidate gets one vote, the candidate the chair votes for wins.

  Otherwise, a candidate would need two out of three votes to win the election.

  Why is this paradoxical? Because the chair had the most power, but the eventual winner of the election was his least-preferred candidate!

Chair	You	Me
A	B	C
B	C	A
C	A	B