Chapter 9: Social Choice: The Impossible Dream

Lesson Plan

- An Introduction to Social Choice
- Majority Rule and Condorcet’s Method
- Other Voting Systems for Three or More Candidates
  - Plurality Voting
  - Borda Count
  - Sequential Pairwise Voting
  - Hare System
- Insurmountable Difficulties: Arrow’s Impossibility Theorem
- A Better Approach? Approval Voting
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An Introduction to Social Choice

Social Choice Theory
- Social choice deals with how groups can best arrive at decisions.
- The problem with social choice is finding good procedures that will turn individual preferences for different candidates into a single choice by the whole group.
- Example: Selecting a winner of an election using a good procedure that will result in an outcome that “reflects the will of the people.”

Preference List Ballot
- A preference list ballot consists of a rank ordering of candidates showing the preferences of one of the individuals who is voting.
- A vertical list is used with the most preferred candidate on top and the least preferred on the bottom.

Throughout the chapter, we assume the number of voters is odd (to help simplify and focus on the theory). Furthermore, in the real world, the number of voters is often so large that ties seldom occur.
Majority Rule

- Majority rule for elections with only two candidates (and an odd number of voters) is a voting system in which the candidate preferred by more than half the voters is the winner.

Three Desirable Properties of Majority Rule

- All voters are treated equally.
- Both candidates are treated equally.
- It is monotone.

Monotone means that if a new election were held and a single voter were to change his/her ballot from voting for the losing candidate to voting for the winning candidate (and everyone else voted the same), the outcome would be the same.

May’s Theorem – Among all two-candidate voting systems that never result in a tie, majority rule is the only one that treats all voters and both candidates equally and is monotone.
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Majority Rule and Condorcet’s Method

Condorcet’s Method

- This method requires that each candidate go head-to-head with each of the other candidates.
- For the two candidates in each contest, record who would win (using majority rule) from each ballot cast. To satisfy Condorcet, the winning candidate must defeat every other candidate one-on-one.
- The Marquis de Condorcet (1743–1794) was the first to realize the voting paradox: If A is better than B, and B is better than C, then A must be better than C. Sometimes C is better than A—not logical!

Condorcet’s Voting Paradox – With three or more candidates, there are elections in which Condorcet’s method yields no winners.
A beats B, 2 out of 3; and
B beats C, 2 out of 3; and
C beats A, 2 out of 3 — No winner!

<table>
<thead>
<tr>
<th>Rank</th>
<th>Number of Voters (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A         B  C</td>
</tr>
<tr>
<td>Second</td>
<td>B         C  A</td>
</tr>
<tr>
<td>Third</td>
<td>C         A  B</td>
</tr>
</tbody>
</table>
Other Voting Systems for Three or More Candidates

When there are three or more candidates, it is more unlikely to have a candidate win with a majority vote. Many other voting methods exist, consisting of reasonable ways to choose a winner; however, they all have shortcomings. We will examine four more popular voting systems for three or more candidates:

Four voting systems, along with their shortcomings:
1. Plurality Voting and the Condorcet Winning Criterion
2. The Borda Count and Independence of Irrelevant Alternatives
3. Sequential Pairwise Voting and the Pareto Condition
4. The Hare System and Monotonicity
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Plurality Voting and the Condorcet Winning Criterion

**Plurality Voting**
- Only first-place votes are considered.
- Even if a preference list ballot is submitted, only the voters’ first choice will be counted—it could have just been a single vote cast.
- The candidate with the most votes wins.
  - The winner does not need a majority of votes, but simply has more votes than the other candidates.

**Example:** Find the plurality vote of the 3 candidates and 13 voters.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Number of Voters (13)</th>
<th>First Place Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

The candidate with the most first-place votes wins. Count each candidate’s first-place votes only. *(A has the most.)*

- A = 5, B = 4, C = 4
- **A wins.**
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Plurality Voting and the Condorcet Winning Criterion

Example: 2000 Presidential Election (Plurality fails CWC.)

- Condorcet Winner Criterion (CWC) is satisfied if either is true:
  1. if there is no Condorcet winner (often the case), or
  2. if the winner of the election is also the Condorcet winner.

This election came down to which of Bush or Gore would carry Florida. Result: George W. Bush won by a few hundred votes.

Gore, however, was considered the Condorcet winner.

It is assumed if Al Gore was pitted against any one of the other three candidates (Bush, Buchanan, Nader), Gore would have won.

Manipulability occurs when voters misrepresent their preference rather than “throw away” their vote.
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Borda Count and Independence of Irrelevant Alternatives

The Borda Count
- Borda Count is a rank method of voting that assigns points in a nonincreasing manner to the ordered candidates on each voter’s preference list ballot and then adds these points to arrive at a group’s final ranking.
- For \( n \) candidates, assign points as follows:
  - First-place vote is worth \( n - 1 \) points, second-place vote is worth \( n - 2 \) points, and so on down to… last-place vote is worth \( n - n = 0 \), zero points.
- The candidate’s total points are referred to as his/her Borda score.

Example: Total the Borda score of each candidate.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Number of Voters (5)</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A A A B B</td>
<td>2</td>
</tr>
<tr>
<td>Second</td>
<td>B B B C C</td>
<td>1</td>
</tr>
<tr>
<td>Third</td>
<td>C C C A A</td>
<td>0</td>
</tr>
</tbody>
</table>

A = 2 + 2 + 2 + 0 + 0 = 6
B = 1 + 1 + 1 + 2 + 2 = 7
C = 0 + 0 + 0 + 1 + 1 = 2

B has the most, B wins.

Another way: Count the occurrences of letters below the candidate—for example, there are 7 “boxes” below B.
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Borda Count and Independence of Irrelevant Alternatives

- **Independence of Irrelevant Alternatives (Borda fails IIA.)**
  - A voting system is said to satisfy independence of irrelevant alternatives (IIA) if it is impossible for candidate B to move from non-winner status to winner status unless at least one voter reverses the order in which he or she had B and the winning candidate ranked.
  - If B was a loser, B should never become a winner, unless he moves ahead of the winner (reverses order) in a voter’s preference list.

**Example showing that Borda count fails to satisfy IIA:** B went from loser to winner and did not switch with A!

Original Borda Score: \(A=6, \ B=5, \ C=4\)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Number of Voters (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A A A C C</td>
</tr>
<tr>
<td>Second</td>
<td>B B B B B</td>
</tr>
<tr>
<td>Third</td>
<td>C C C A A</td>
</tr>
</tbody>
</table>

Suppose the last two voters change their ballots (reverse the order of just the losers). This should **not** change the winner.

New Borda Score: \(A=6, \ B=7, \ C=2\)

<table>
<thead>
<tr>
<th>Rank</th>
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</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A A A A B B</td>
</tr>
<tr>
<td>Second</td>
<td>B B B B C C</td>
</tr>
<tr>
<td>Third</td>
<td>C C C C A A</td>
</tr>
</tbody>
</table>
Sequential Pairwise Voting

Sequential pairwise voting starts with an agenda and pits the first candidate against the second in a one-on-one contest.

The losers are deleted and the winner then moves on to confront the third candidate in the list, one on one.

This process continues throughout the entire agenda, and the one remaining at the end wins.

Example: Who would be the winner using the agenda A, B, C, D for the following preference list ballots of three voters?

<table>
<thead>
<tr>
<th>Rank</th>
<th>Number of Voters (3)</th>
<th>Preference List</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A C B</td>
<td>A vs. B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A vs. C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C vs. D</td>
</tr>
<tr>
<td>Second</td>
<td>B A D</td>
<td>A wins; B is</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C wins; A is</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D wins; C is</td>
</tr>
<tr>
<td>Third</td>
<td>D B C</td>
<td>deleted.</td>
</tr>
<tr>
<td>Fourth</td>
<td>C D A</td>
<td>deleted.</td>
</tr>
</tbody>
</table>

Using the agenda A, B, C, D, start with A vs. B and record (with tally marks) who is preferred for each ballot list (column).

Candidate D wins for this agenda.
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Sequential Pairwise Voting and the Pareto Condition

- **Pareto Condition** (Sequential Pairwise fails Pareto.)
  - Pareto condition states that if everyone prefers one candidate (in this case, B) to another candidate (D), then this latter candidate (D) should not be among the winners of the election.
  - Pareto condition is named after Vilfredo Pareto (1848–1923), Italian economist.

- From the last example:
  - D was the winner for the agenda A, B, C, D.
  - However, each voter (each of the three preference lists columns) preferred B over D.
  - If everyone preferred B to D, then D should not have been the winner! *Not fair!*

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</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>A C B</td>
</tr>
<tr>
<td>Second</td>
<td>B A D</td>
</tr>
<tr>
<td>Third</td>
<td>D B C</td>
</tr>
<tr>
<td>Fourth</td>
<td>C D A</td>
</tr>
</tbody>
</table>

Different agenda orders can change the outcomes. For example, agenda D, C, B, A results in A as the winner.
The Hare System

- The Hare system proceeds to arrive at a winner by repeatedly deleting candidates that are “least preferred” (meaning at the top of the fewest ballots).
- If a single candidate remains after all others have been eliminated, he/she alone is the winner.
- If two or more candidates remain and they all would be eliminated in the next round, then these candidates would tie.

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<tbody>
<tr>
<td>First</td>
<td>A  C  B  B</td>
</tr>
<tr>
<td>Second</td>
<td>B  B  C  A</td>
</tr>
<tr>
<td>Third</td>
<td>C  A  A  C</td>
</tr>
</tbody>
</table>

For the Hare system, delete the candidate with the least first-place votes:

A = 5, B = 4, and C = 4

Since B and C are tied for the least first-place votes, they are both deleted and A wins.
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The Hare System and Monotonicity

- **Monotonicity** (The Hare system fails monotonicity.)
  - Monotonicity says that if a candidate is a winner and a new election is held in which the only ballot change made is for some voter to move the former winning candidate higher on his/her ballot, then the original winner should remain a winner.
  - In a new election, if a voter moves a winner higher up on his preference list, the outcome should still have the same winner.

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</tr>
<tr>
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<td>C</td>
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</table>

In the previous example, A won. For the last voter, move A up higher on the list (A and B switch places). Round 1: B is deleted. Round 2: C moves up to replace B on the third column. However, C wins—this is a glaring defect!
Insurmountable Difficulties: Arrow’s Impossibility Theorem

Arrow’s Impossibility Theorem

Kenneth Arrow, an economist in 1951, proved that finding an absolutely fair and decisive voting system is impossible.

With three or more candidates and any number of voters, there does not exist—and there never will exist—a voting system that always produces a winner, satisfies the Pareto condition and independence of irrelevant alternatives (IIA), and is not a dictatorship.

If you had an odd number of voters, there does not exist—and there never will exist—a voting system that satisfies both the CWC and IIA and that always produces at least one winner in every election.
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A Better Approach? Approval Voting

**Approval Voting**

- Under approval voting, each voter is allowed to give one vote to as many of the candidates as he or she finds acceptable.
- No limit is set on the number of candidates for whom an individual can vote; however, preferences cannot be expressed.
- Voters show disapproval of other candidates simply by not voting for them.
- The winner under approval voting is the candidate who receives the largest number of approval votes.
- This approach is also appropriate in situations where more than one candidate can win, for example, in electing new members to an exclusive society such as the National Academy of Sciences or the Baseball Hall of Fame. Approval voting is also used to elect the secretary general of the United Nations.
- Approval voting was proposed independently by several analysts in the 1970s.