Inventory Management

12

PowerPoint presentation to accompany
Heizer, Render, Munson
Operations Management, Twelfth Edition
Principles of Operations Management, Tenth Edition

PowerPoint slides by Jeff Heyl

Inventory Models for Independent Demand

Need to determine when and how much to order

1. Basic economic order quantity (EOQ) model
2. Production order quantity model
3. Quantity discount model

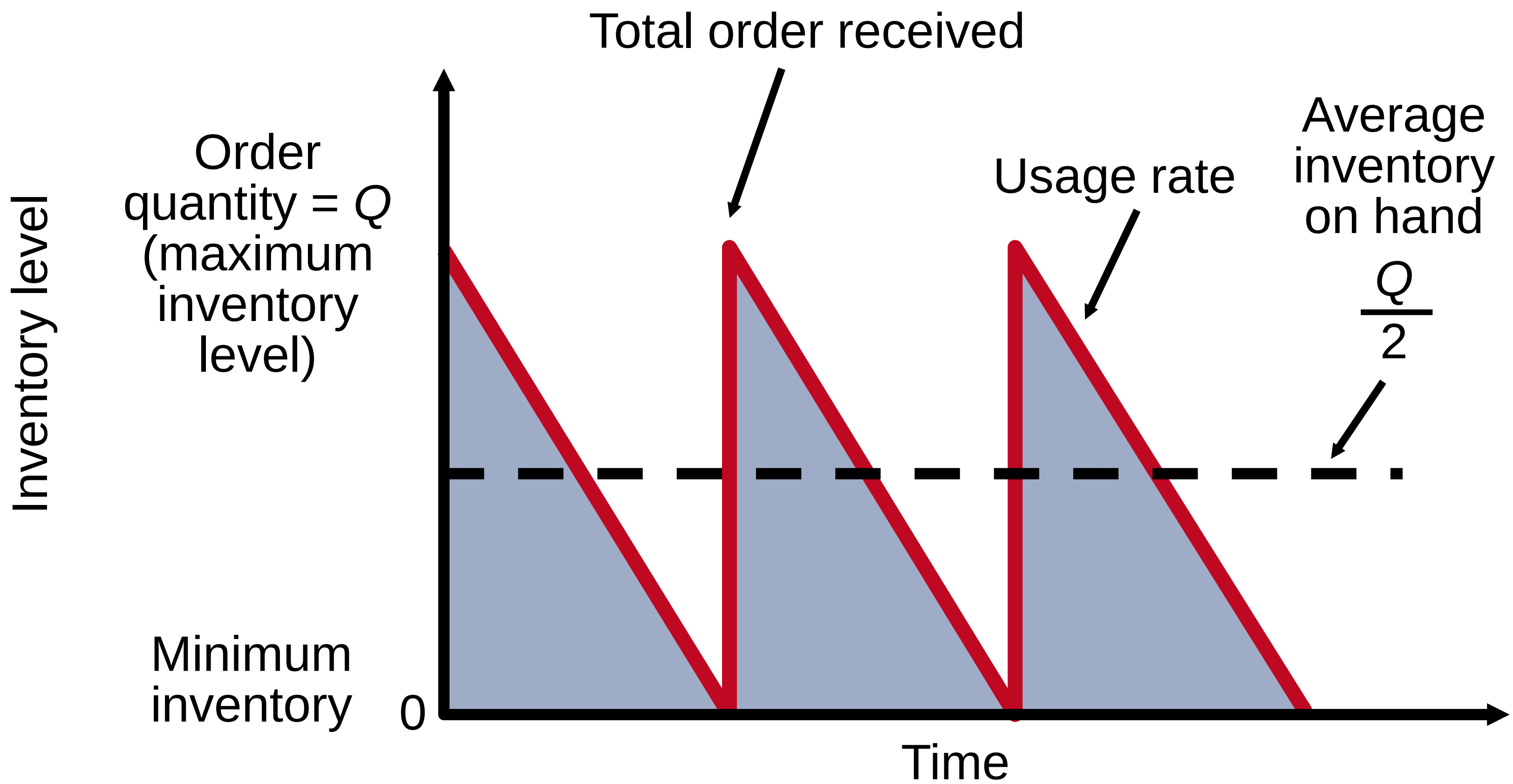
Basic EOQ Model

Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup (or ordering) and holding
6. Stockouts can be completely avoided

Inventory Usage Over Time

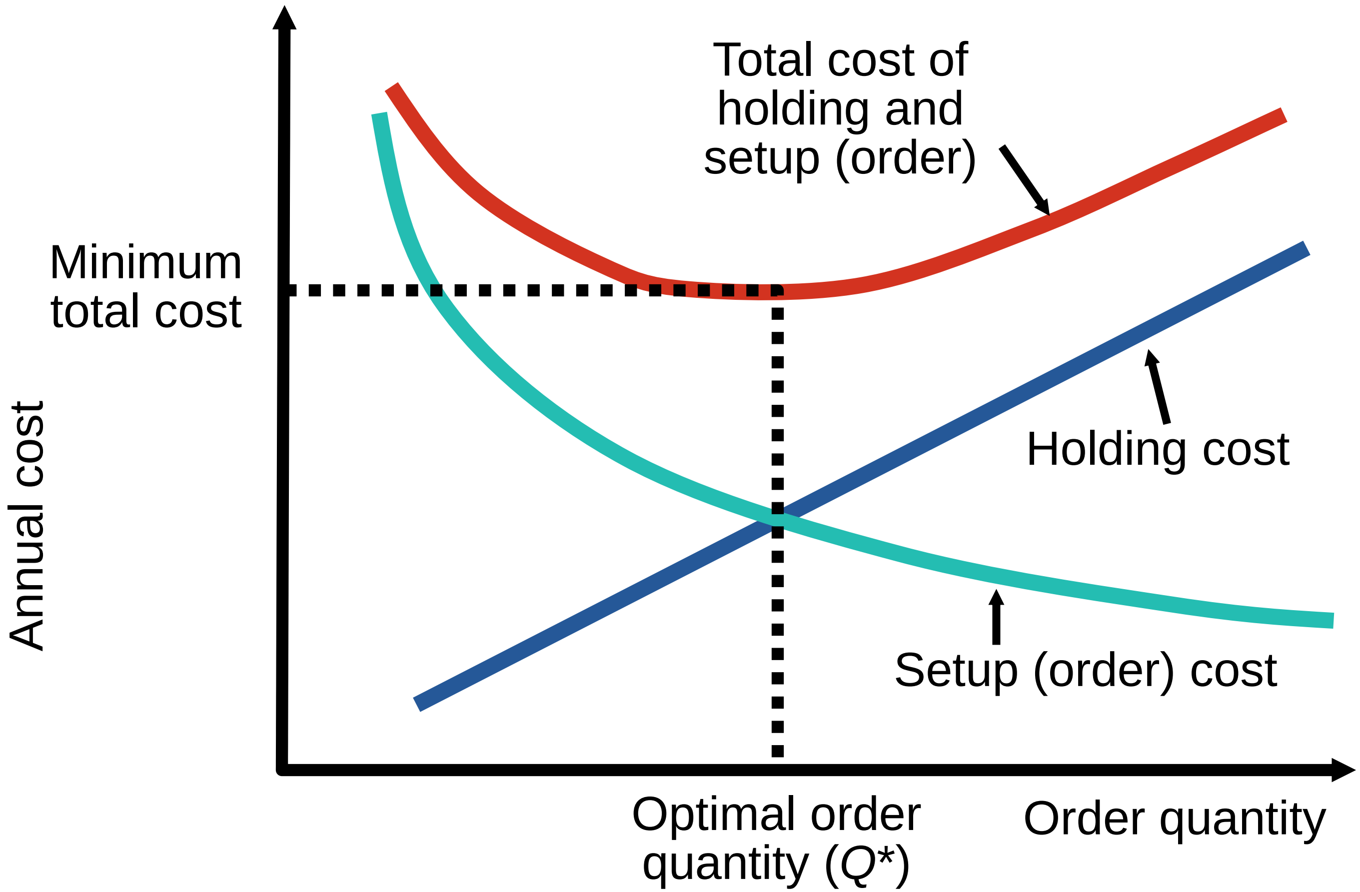
Figure 12.3



Minimizing Costs

Objective is to minimize total costs

Table 12.4(c)



Minimizing Costs

- By minimizing the sum of setup (or ordering) and holding costs, total costs are minimized
- Optimal order size Q^* will minimize total cost
- A reduction in either cost reduces the total cost
- Optimal order quantity occurs when holding cost and setup cost are equal

Minimizing Costs

Q = Number of units per order

Q^* = Optimal number of units per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year) x (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\begin{array}{c}\text{Setup or order}\\ \text{cost per order}\end{array}\right)$$

$$= \left(\frac{D}{Q}\right)S$$

Minimizing C

$$\text{Annual setup cost} = \frac{D}{Q}S$$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year) x (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\begin{array}{c}\text{Setup or order}\\ \text{cost per order}\end{array}\right)$$

$$= \left(\frac{D}{Q}\right)S$$

Minimizing C

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Annual holding cost = (Average inventory level)
x (Holding cost per unit per year)

$$= \left(\frac{\text{Order quantity}}{2}\right)\text{(Holding cost per unit per year)}$$

$$= \left(\frac{Q}{2}\right)H$$

Minimizing C

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\left(\frac{D}{Q}\right)S = \left(\frac{Q}{2}\right)H$$

Solving for Q^*

$$2DS = Q^2 H$$

$$Q^2 = \frac{2DS}{H}$$

$$Q^* = \sqrt{\frac{2DS}{H}}$$

An EOQ Example

Determine **optimal number of needles to order**

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ Example

Determine **expected number of orders**

D = 1,000 units $\qquad$ Q^* = 200 units

S = $10 per order

H = $.50 per unit per year

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = \text{5 orders per year}$$

An EOQ Example

Determine **optimal time between orders**

D = 1,000 units

S = $10 per order

H = $.50 per unit per year

Q^* = 200 units

N = 5 orders/year

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{\text{Expected number of orders}}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine the **total annual cost**

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 200 units

N = 5 orders/year

T = 50 days

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= (5)(\$10) + (100)(\$.50)$$

$$= \$50 + \$50 = \$100$$

The EOQ Model

When including actual cost of material P

Total annual cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Robust Model

- The EOQ model is **robust**
- It works even if all parameters and assumptions are not met
- The total cost curve is relatively flat in the area of the EOQ

An EOQ Example

Determine optimal number of needles to order

D = ~~1,000 units~~ 1,500 units

S = \$10 per order

H = \$.50 per unit per year

N= 5 orders/year

$Q^*_{1,000}$ = 200 units

T = 50 days

$Q^*_{1,500}$ = 244.9 units

Ordering old *Q**

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1,500}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= \$75 + \$50 = \$125$$

Ordering new *Q**

$$= \frac{1,500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$= 6.125(\$10) + 122.45(\$.50)$$

$$= \$61.25 + \$61.22 = \$122.47$$

An EOQ Example

Determine optimal number of

D = ~~1,000 units~~ 1,500 units

S = $10 per order

H = $.50 per unit per year

N= 5 orders/year

Only 2% less than the total cost of $125 when the order quantity was 200

Ordering old Q*

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= \$75 + \$50 = \$125$$

Or

$$= \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$= 6.125(\$10) + 122.45(\$.50)$$

$$= \$61.25 + \$61.22 = \$122.47$$

Reorder Points

- EOQ answers the "how much" question
- The reorder point (ROP) tells "when" to order
- Lead time (L) is the time between placing and receiving an order

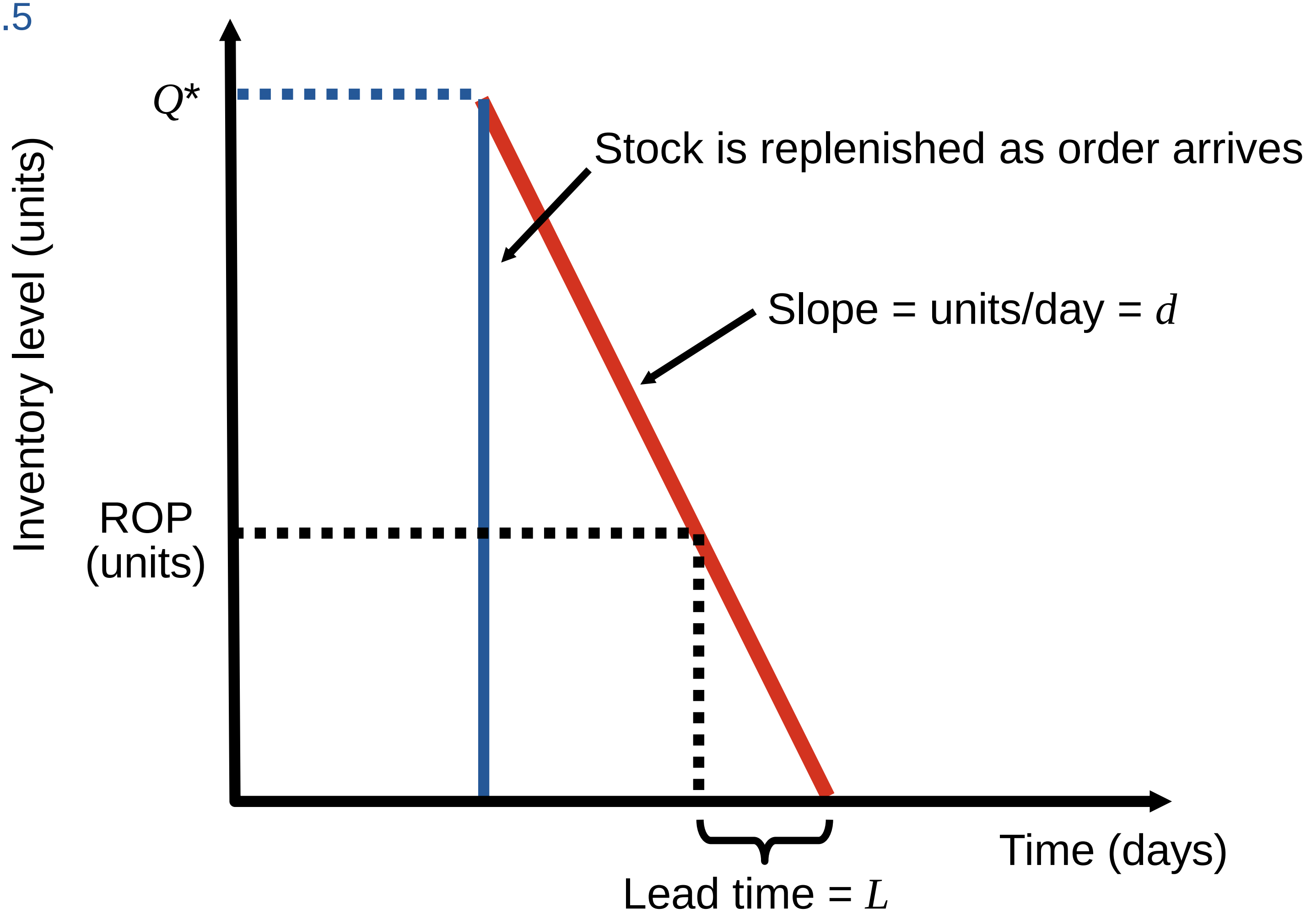
$$\text{ROP} = \left(\begin{array}{c}\text{Demand}\\ \text{per day}\end{array}\right)\left(\begin{array}{c}\text{Lead time for a new}\\ \text{order in days}\end{array}\right)$$

$$\text{ROP} = d \text{ x } L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve

Figure 12.5



Reorder Point Example

Demand = 8,000 iPhones per year
250 working day year
Lead time for orders is 3 working days, may take 4

$$d = \frac{D}{\text{Number of working days in a year}}$$

$$= 8{,}000/250 = 32 \text{ units}$$

$$\text{ROP} = d \times L$$

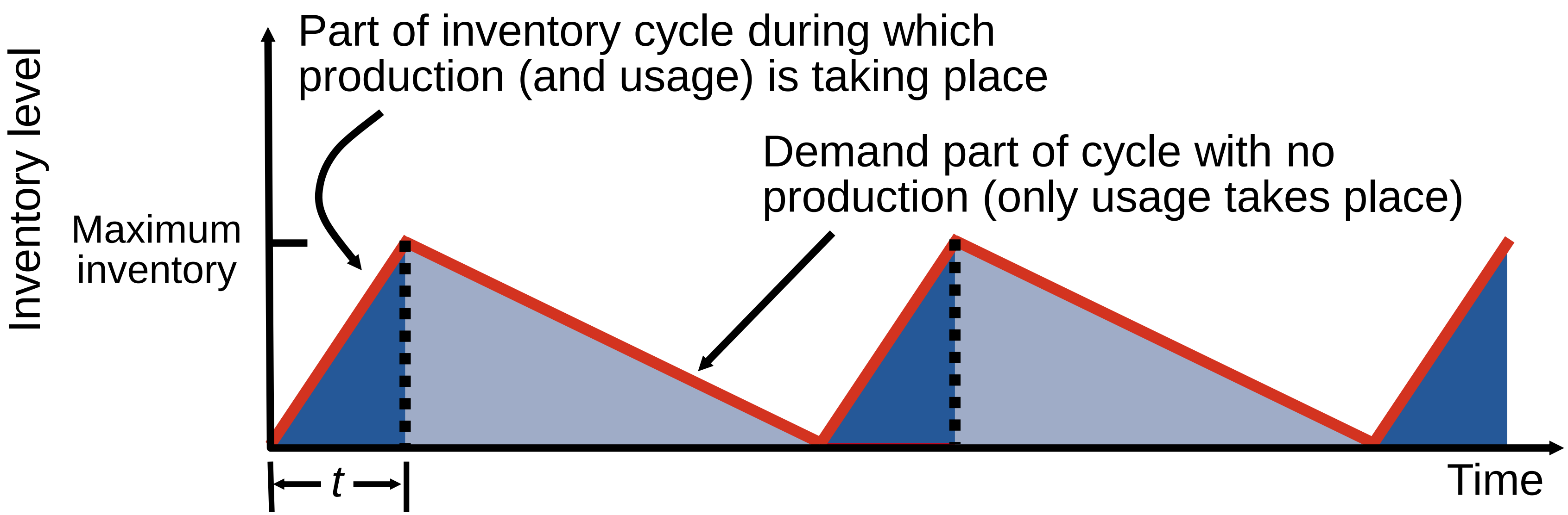
$$= 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

$$= 32 \text{ units per day} \times 4 \text{ days} = 128 \text{ units}$$

Production Order Quantity Model

1. Used when inventory builds up over a period of time after an order is placed
2. Used when units are produced and sold simultaneously

Figure 12.6



Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Annual inventory}\\ \text{holding cost}\end{pmatrix} = (\text{Average inventory level}) \times \begin{pmatrix}\text{Holding cost}\\ \text{per unit per year}\end{pmatrix}$$

$$\begin{pmatrix}\text{Annual inventory}\\ \text{level}\end{pmatrix} = (\text{Maximum inventory level})/2$$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)H\right]$$

Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\text{Setup cost} = (D/Q)S$$

$$\text{Holding cost} = \tfrac{1}{2}HQ\left[1-\left(d/p\right)\right]$$

$$\frac{D}{Q}S = \tfrac{1}{2}HQ\left[1-\left(d/p\right)\right]$$

$$Q^2 = \frac{2DS}{H\left[1-\left(d/p\right)\right]}$$

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1-\left(d/p\right)\right]}}$$

Production Order Quantity Example

D = 1,000 units
S = \$10
H = \$0.50 per unit per year

p = 8 units per day
d = 4 units per day

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1 - \left(d/p\right)\right]}}$$

$$Q_p^* = \sqrt{\frac{2(1{,}000)(10)}{0.50\left[1 - (4/8)\right]}}$$

$$= \sqrt{\frac{20{,}000}{0.50(1/2)}} = \sqrt{80{,}000}$$

$$= 282.8 \text{ hubcaps, or } 283 \text{ hubcaps}$$

Production Order Quantity Model

Note:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1{,}000}{250}$$

When annual data are used the equation becomes:

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \frac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}}$$

Quantity Discount Models

- Reduced prices are often available when larger quantities are purchased
- Trade-off is between reduced product cost and increased holding cost

TABLE 12.2	A Quantity Discount Schedule	
PRICE RANGE	**QUANTITY ORDERED**	**PRICE PER UNIT *P***
Initial price	0 to 119	$100
Discount price 1	200 to 1,499	$ 98
Discount price 2	1,500 and over	$ 96

Quantity Discount Models

Total annual cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + PD$$

where Q = Quantity ordered
D = Annual demand in units
S = Ordering or setup cost per order

P = Price per unit
I = Holding cost per unit per year expressed as a percent of price P

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

Because unit price varies, holding cost is expressed as a percent (I) of unit price (P)

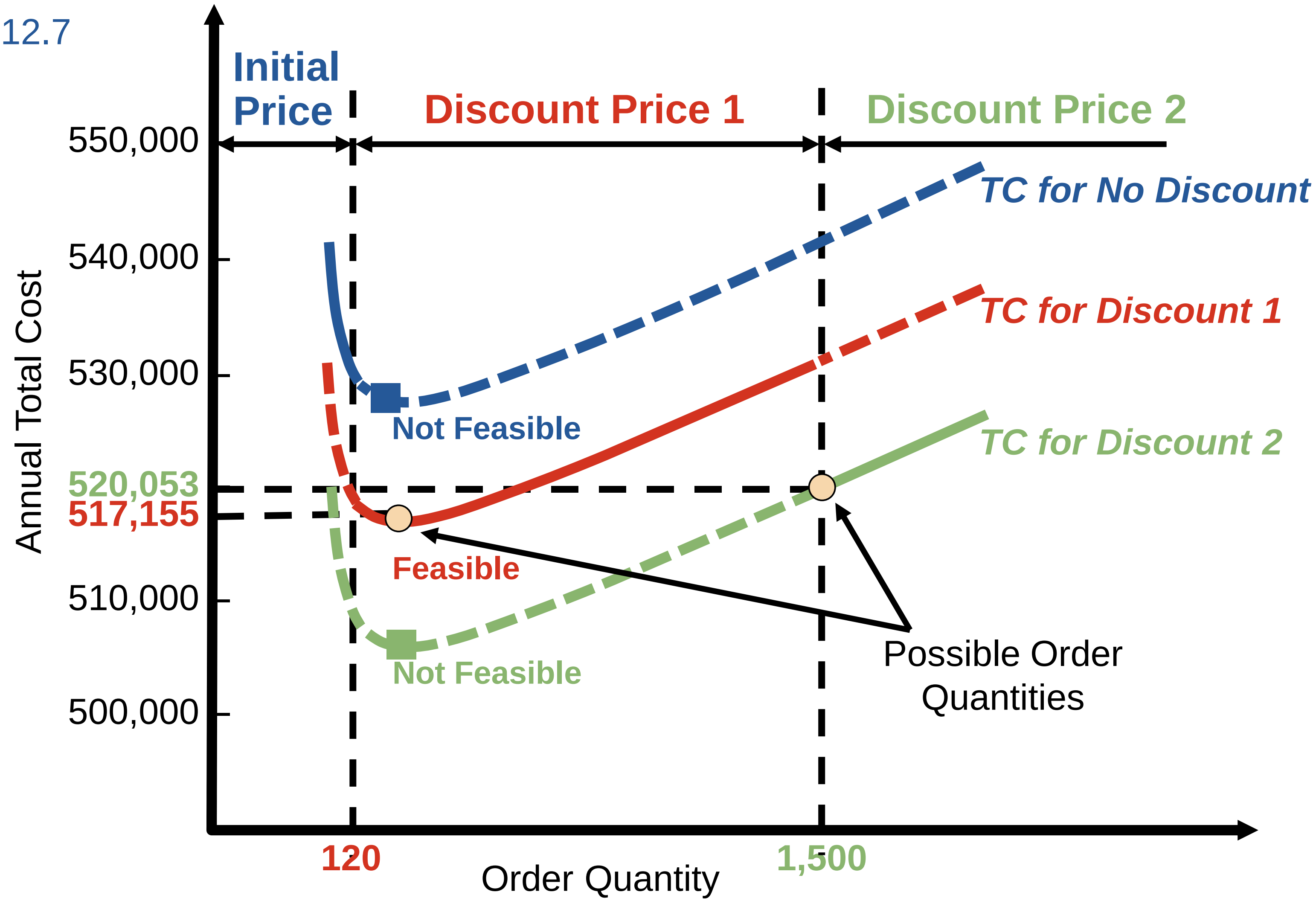
Quantity Discount Models

Steps in analyzing a quantity discount

1. Starting with the *lowest* possible purchase price, calculate Q^* until the first feasible EOQ is found. This is a possible best order quantity, along with all price-break quantities for all *lower* prices.
2. Calculate the total annual cost for each possible order quantity determined in Step 1. Select the quantity that gives the lowest total cost.

Quantity Discount Models

Figure 12.7



Quantity Discount Example

Calculate Q^* for every discount starting with the lowest price

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{\$96}^* = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$96)}} = \cancel{278} \text{ drones/order}$$

Infeasible – calculate Q* for next-higher price

$$Q_{\$98}^* = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$98)}} = 275 \text{ drones/order}$$

Feasible

Quantity Discount Example

TABLE 12.3	Total Cost Computations for Chris Beehner Electronics				
ORDER QUANTITY	UNIT PRICE	ANNUAL ORDERING COST	ANNUAL HOLDING COST	ANNUAL PRODUCT COST	TOTAL ANNUAL COST
275	$98	$3,782	$3,773	$509,600	$517,155
1,500	$96	$693	$20,160	$499,200	$520,053

Choose the price and quantity that gives the lowest total cost

Buy 275 drones at $98 per unit

Quantity Discount Variations

- *All-units discount* is the most popular form
- *Incremental quantity discounts* apply only to those units purchased beyond the price break quantity
- *Fixed fees* may encourage larger purchases
- *Aggregation* over items or time
- *Truckload discounts*, *buy-one-get-one-free offers*, *one-time-only sales*

Probabilistic Models and Safety Stock

- Used when demand is not constant or certain
- Use safety stock to achieve a desired **service level** and avoid stockouts

$$\text{ROP} = d \times L + ss$$

Annual stockout costs = **The sum of the units short for each demand level** x **The probability of that demand level** x **The stockout cost/unit** x **The number of orders per year**

Safety Stock Example

ROP = 50 units
Orders per year = 6

Stockout cost = $40 per frame
Carrying cost = $5 per frame per year

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP → 50	.3
60	.2
70	.1
	1.0

Safety Stock Example

ROP = 50 units
Orders per year = 6

Stockout cost = $40 per frame
Carrying cost = $5 per frame per year

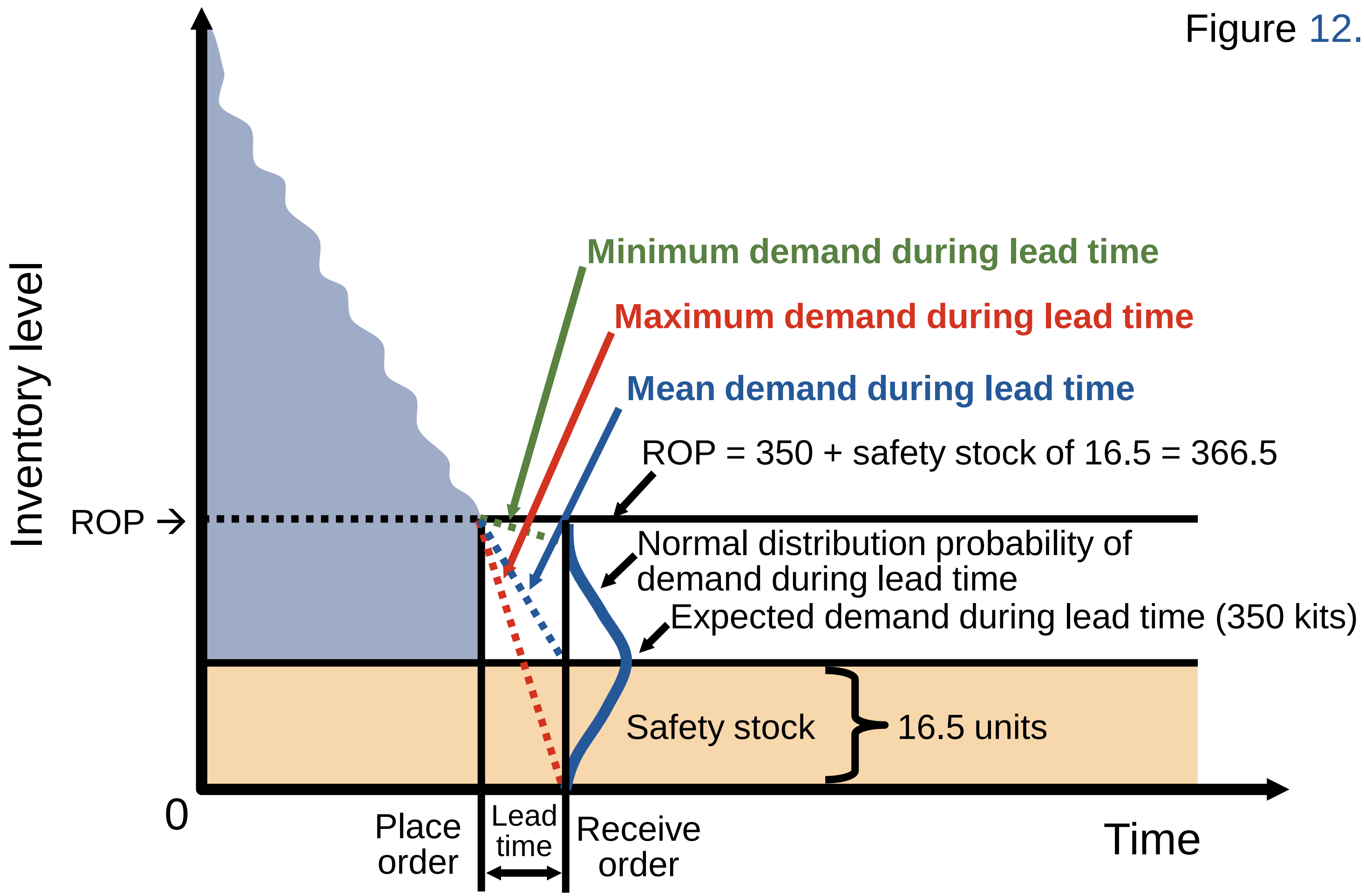
SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	(20)($5) = $100	$0	$100
10	(10)($5) = $ 50	(10)(.1)($40)(6) = $240	$290
0	$ 0	(10)(.2)($40)(6) + (20)(.1)($40)(6) = $960	$960

A safety stock of 20 frames gives the lowest total cost

ROP = 50 + 20 = 70 frames

Probabilistic Demand

Figure 12.8



Probabilistic Demand

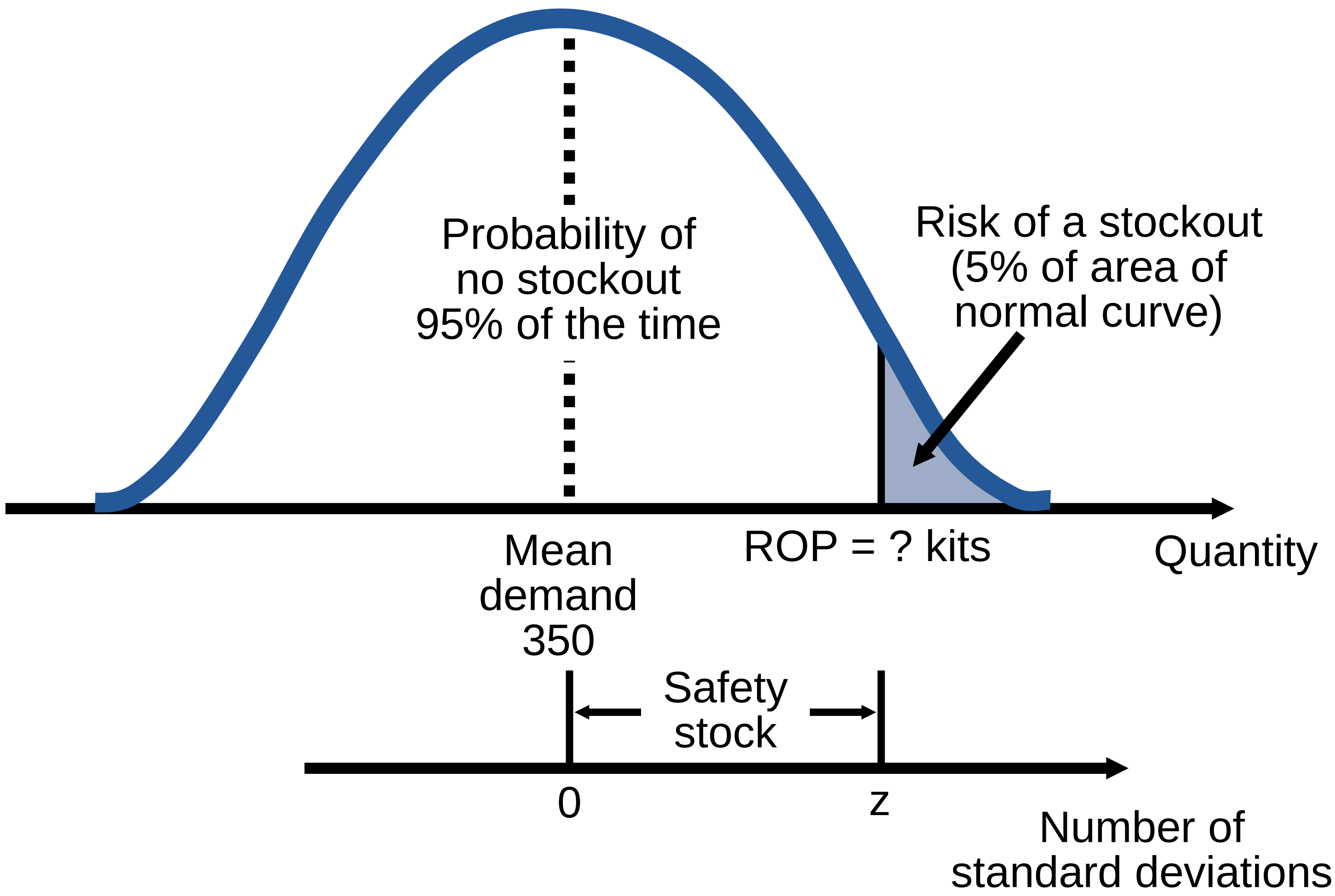
Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

$$ROP = \text{demand during lead time} + Z\sigma_{dLT}$$

where Z = Number of standard deviations

σ_{dLT} = Standard deviation of demand during lead time

Probabilistic Demand



Probabilistic Example

μ = Average demand = 350 kits

σ_{dLT} = Standard deviation of demand during lead time = 10 kits

Stockout policy = 5% (service level = 95%)

Using Appendix I, for an area under the curve of 95%, the Z = 1.645

Safety stock = $Z\sigma_{dLT}$ = 1.645(10) = 16.5 kits

Reorder point = Expected demand during lead time + Safety stock

= 350 kits + 16.5 kits of safety stock

= 366.5 or 367 kits

Other Probabilistic Models

- When data on demand during lead time is not available, there are other models available
  1. When demand is variable and lead time is constant
  2. When lead time is variable and demand is constant
  3. When both demand and lead time are variable

Other Probabilistic Models

Demand is variable and lead time is constant

ROP = (*Average* daily demand x Lead time in days) + $Z\sigma_{dLT}$

where $\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$

σ_d = Standard deviation of demand per day

Probabilistic Example

Average daily demand (normally distributed) = 15
Lead time in days (constant) = 2
Standard deviation of daily demand = 5
Service level = 90%

Z for 90% = 1.28
From Appendix I

$$\begin{aligned} \text{ROP} &= (15 \text{ units x } 2 \text{ days}) + Z\sigma_{dLT} \\ &= 30 + 1.28(5)(\sqrt{2}) \\ &= 30 + 9.02 = 39.02 \approx 39 \end{aligned}$$

Safety stock is about 9 computers

Other Probabilistic Models

Lead time is variable and demand is constant

ROP = (Daily demand x *Average* lead time in days) + Z x (Daily demand) x σ_{LT}

where σ_{LT} = Standard deviation of lead time in days

Probabilistic Example

Daily demand (constant) = 10
Average lead time = 6 days
Standard deviation of lead time = σ_{LT} = 1
Service level = 98%, so Z (from Appendix I) = 2.055

ROP = (10 units x 6 days) + 2.055(10 units)(1)
= 60 + 20.55 = 80.55

Reorder point is about 81 cameras

Other Probabilistic Models

Both demand and lead time are variable

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time}) + Z\sigma_{dLT}$$

where σ_d = Standard deviation of demand per day

σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma^2_{LT}}$$

Probabilistic Example

Average daily demand (normally distributed) = 150
Standard deviation = σ_d = 16
Average lead time 5 days (normally distributed)
Standard deviation = σ_{LT} = 1 day
Service level = 95%, so Z = 1.645 (from Appendix I)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.645\sigma_{dLT}$$

$$\sigma_{dLT} = \sqrt{\left(5 \text{ days} \times 16^2\right) + \left(150^2 \times 1^2\right)} = \sqrt{\left(5 \times 256\right) + \left(22{,}500 \times 1\right)}$$

$$= \sqrt{\left(1{,}280\right) + \left(22{,}500\right)} = \sqrt{23{,}780} \cong 154$$

$$\text{ROP} = (150 \times 5) + 1.645(154) \cong 750 + 253 = 1{,}003 \text{ packs}$$

Single-Period Model

- Only *one* order is placed for a product
- Units have little or no value at the end of the sales period

C_s = Cost of shortage = Sales price/unit – Cost/unit

C_o = Cost of overage = Cost/unit – Salvage value

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single-Period Example

Average demand = μ = 120 papers/day

Standard deviation = σ = 15 papers

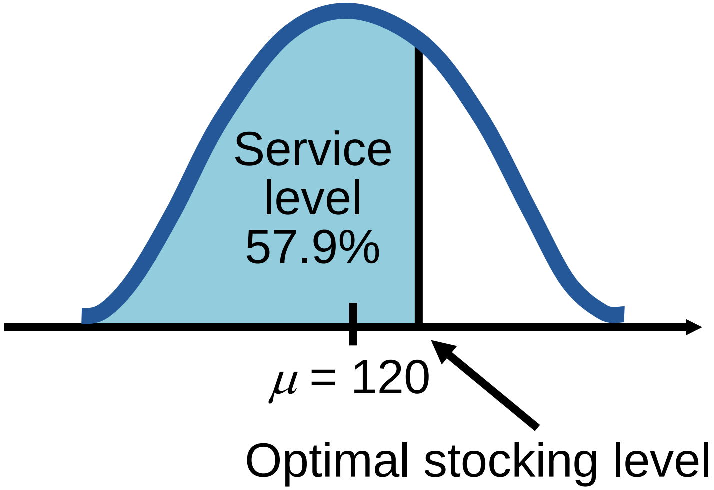
C_s = cost of shortage = $1.25 – $.70 = $.55

C_o = cost of overage = $.70 – $.30 = $.40

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

$$= \frac{.55}{.55 + .40}$$

$$= \frac{.55}{.95} = .579$$



Single-Period Example

From Appendix I, for the area .579, $Z \cong .20$

The optimal stocking level

= 120 copies + (.20)(σ)

= 120 + (.20)(15) = 120 + 3 = 123 papers

The stockout risk = 1 – Service level

= 1 – .579 = .422 = 42.2%

Fixed-Period (*P*) Systems

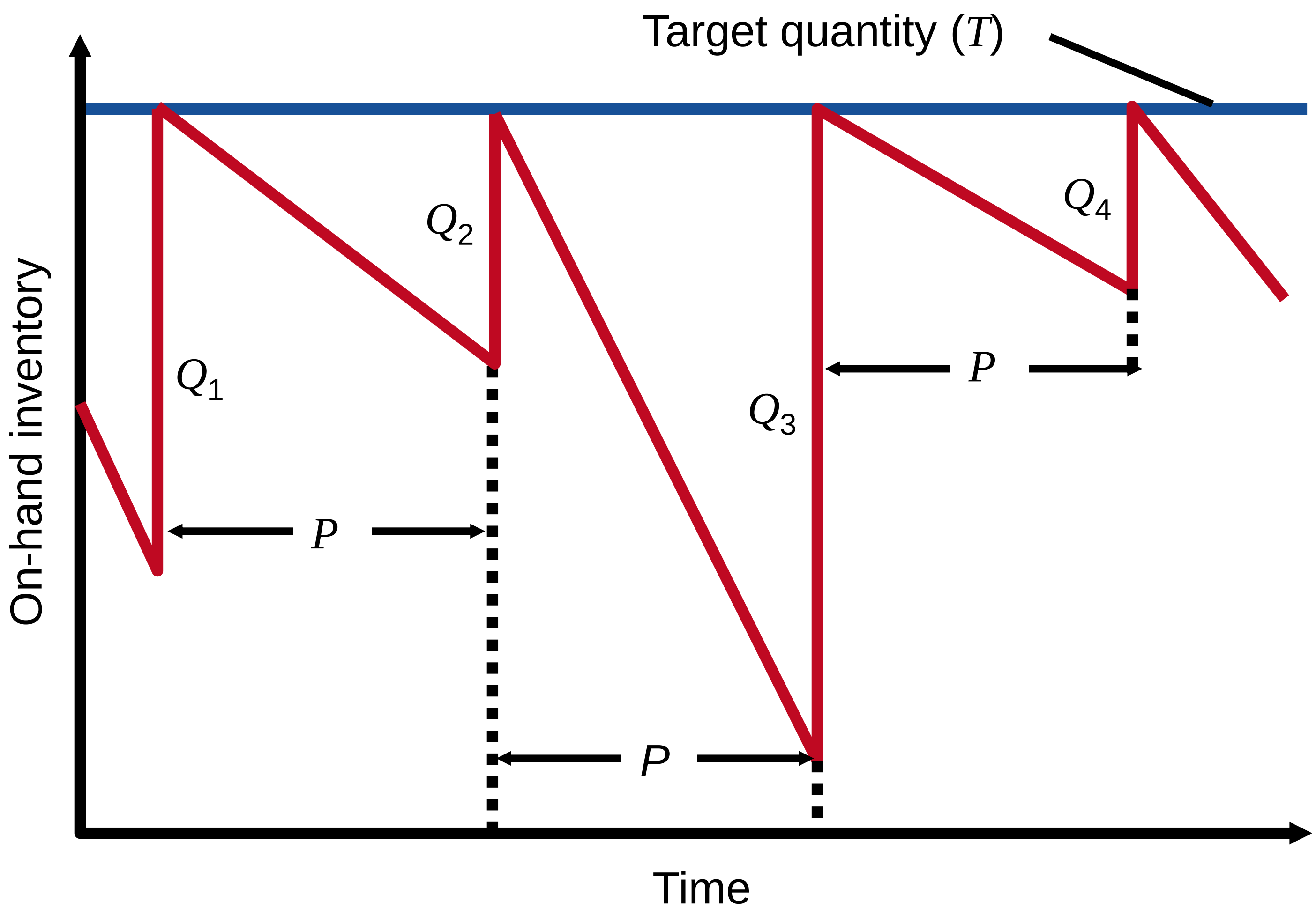
- **Fixed-quantity** models require continuous monitoring using **perpetual inventory systems**
- In **fixed-period systems** orders placed at the end of a fixed period
- Periodic review, ***P* system**

Fixed-Period (*P*) Systems

- Inventory counted only at end of period
- Order brings inventory up to target level
  - Only relevant costs are ordering and holding
  - Lead times are known and constant
  - Items are independent of one another

Fixed-Period (*P*) Systems

Figure 12.9



Fixed-Period Systems

- Inventory is only counted at each review period
- May be scheduled at convenient times
- Appropriate in routine situations
- May result in stockouts between periods
- May require increased safety stock