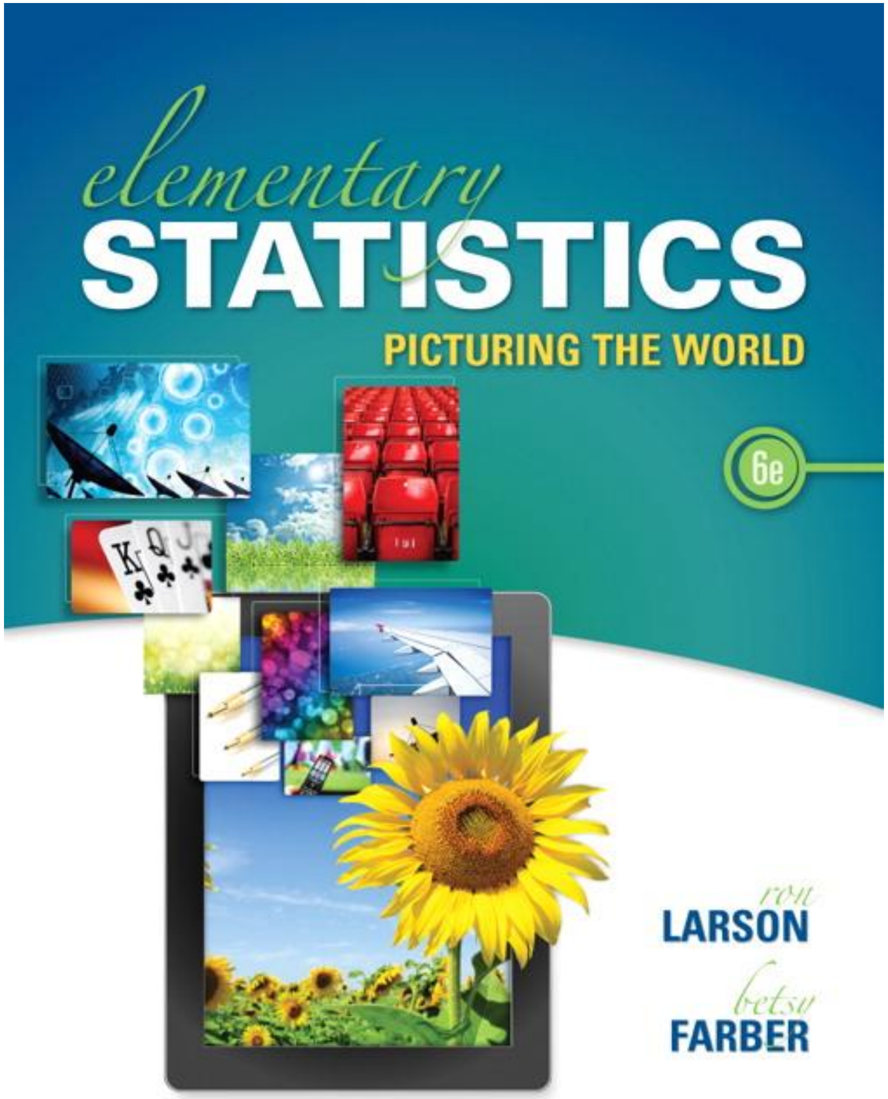
Chapter 6

Confidence Intervals



Point Estimate for Population μ

Point Estimate

- A single value estimate for a population parameter
- Most unbiased point estimate of the population mean μ is the sample mean $\bar{x}$

Estimate Population Parameter…	with Sample Statistic
Mean: μ	$\bar{x}$

Example: Point Estimate for Population μ

An economics researcher is collecting data about grocery store employees in a county. The data listed below represents a random sample of the number of hours worked by 40 employees from several grocery stores in the county. Find a point estimate of the population mean, μ.

30	26	33	26	26	33	31	31	21	37
27	20	34	35	30	24	38	34	39	31
22	30	23	23	31	44	31	33	33	26
27	28	25	35	23	32	29	31	25	27

Solution: Point Estimate for Population *μ*

The sample mean of the data is

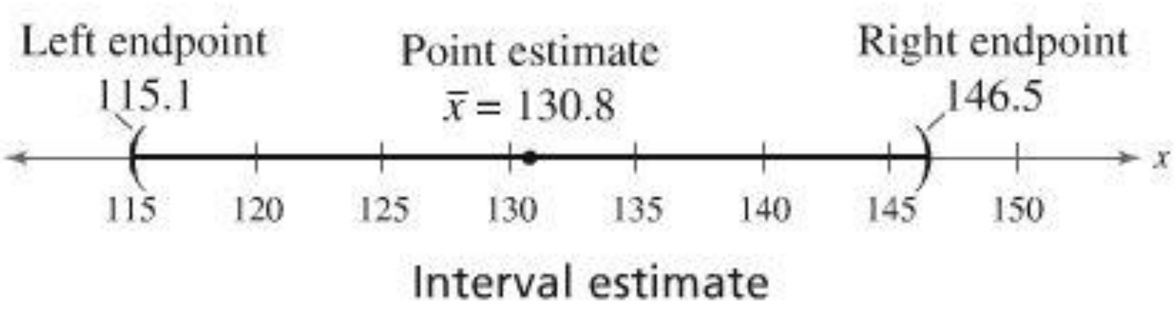
$$\bar{x} = \frac{\Sigma x}{n} = \frac{1184}{40} = 29.6$$

The point estimate for the mean number of hours worked by grocery store employees in this county is 29.6 hours.

Interval Estimate

Interval estimate

- An interval, or range of values, used to estimate a population parameter.

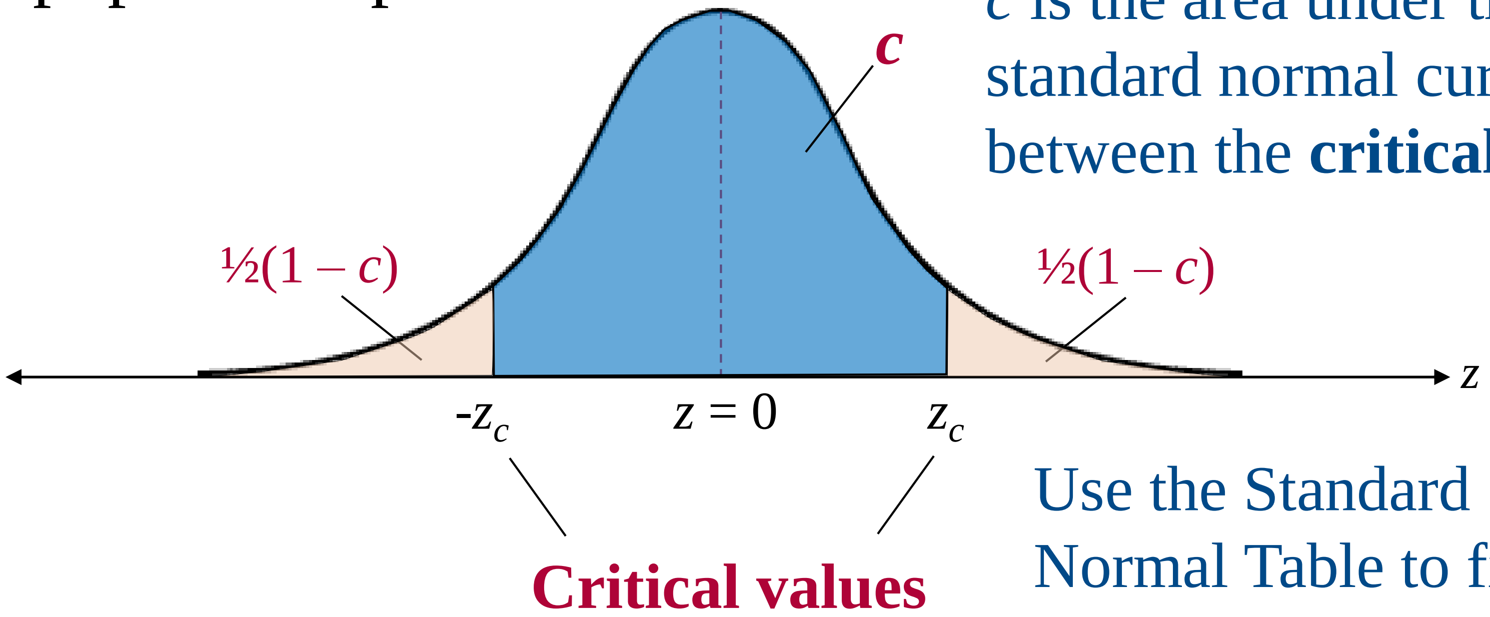


How confident do we want to be that the interval estimate contains the population mean μ?

Level of Confidence

Level of confidence *c*

- The probability that the interval estimate contains the population parameter.



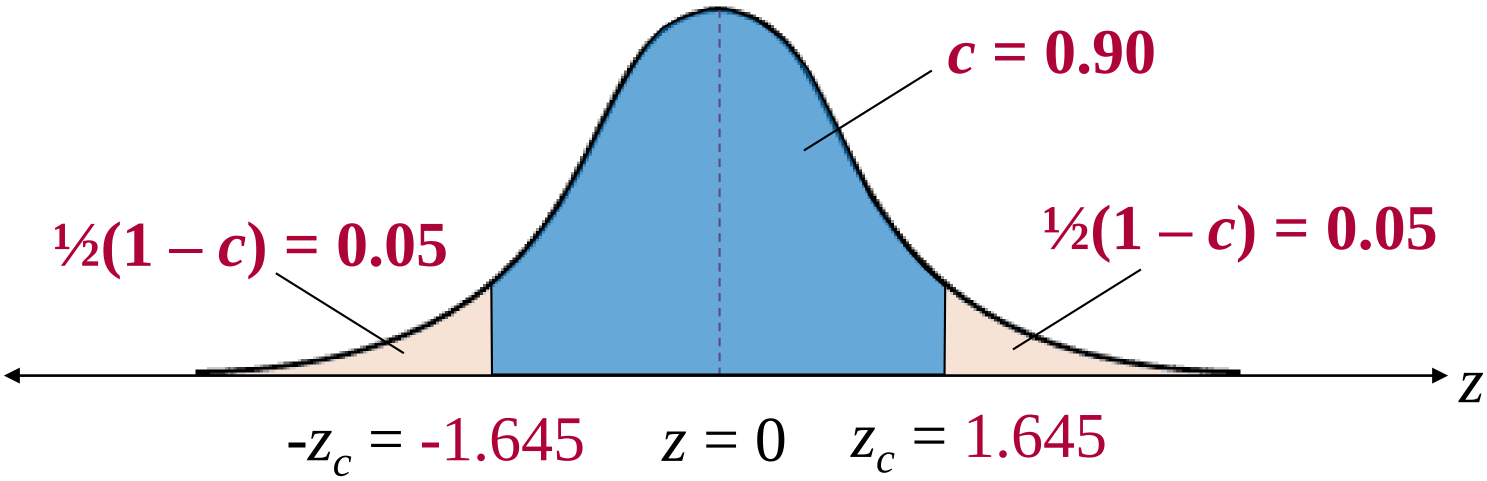
c is the area under the standard normal curve between the **critical values**.

Use the Standard Normal Table to find the corresponding *z*-scores.

The remaining area in the tails is $1 - c$.

Level of Confidence

- If the level of confidence is 90%, this means that we are 90% confident that the interval contains the population mean μ.



The corresponding z-scores are ± 1.645.

Sampling Error

Sampling error

- The difference between the point estimate and the actual population parameter value.
- For μ:
  - the sampling error is the difference $\bar{x} - \mu$
  - μ is generally unknown
  - $\bar{x}$ varies from sample to sample

Margin of Error

Margin of error

- The greatest possible distance between the point estimate and the value of the parameter it is estimating for a given level of confidence, *c*.
- Denoted by *E*.

$$E = z_c \sigma_{\bar{x}} = z_c \frac{\sigma}{\sqrt{n}}$$

- 1. The sample is random.
- 2. Population is normally distributed or $n \geq 30$.

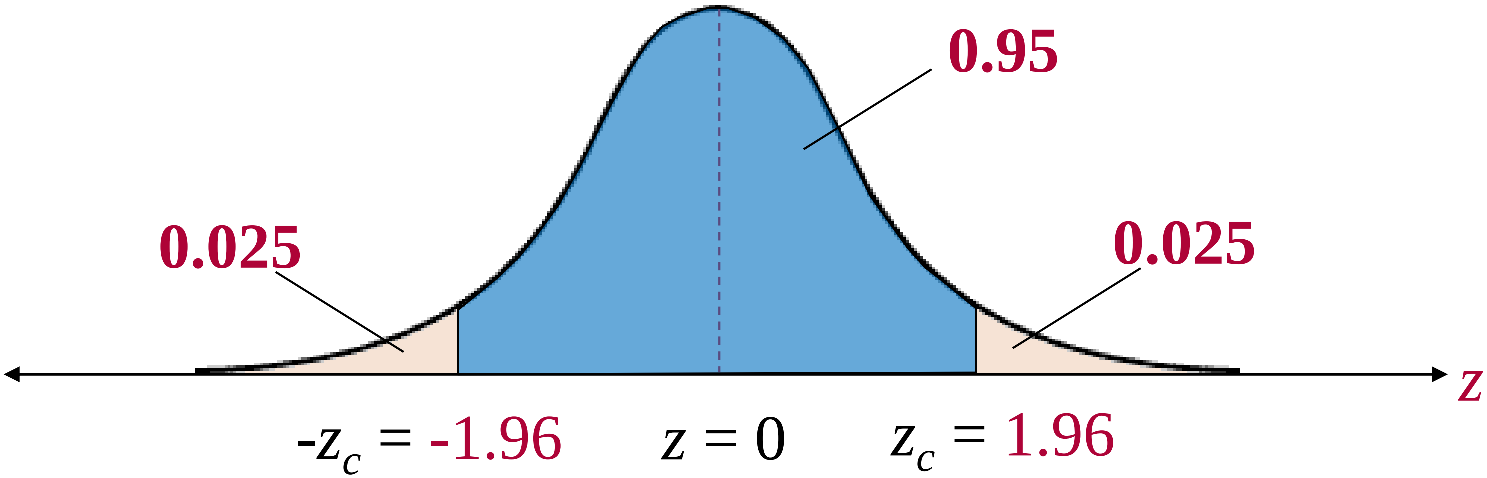
Sometimes called the maximum error of estimate or error tolerance.

Example: Finding the Margin of Error

Use the data about the grocery store employees and a 95% confidence level to find the margin of error for the mean number of hours worked by grocery store employees. Assume the population standard deviation is 7.9 hours.

Solution: Finding the Margin of Error

- First find the critical values



95% of the area under the standard normal curve falls within 1.96 standard deviations of the mean. (You can approximate the distribution of the sample means with a normal curve by the Central Limit Theorem, because $n = 40 \geq 30$.)

Solution: Finding the Margin of Error

$$E = z_c \frac{s}{\sqrt{n}}$$

$$\text{»}\, 1.96 \times \frac{7.9}{\sqrt{40}}$$

$$\text{»}\, 2.4$$

You are 95% confident that the margin of error for the population mean is about 2.4 hours.

Confidence Intervals for the Population Mean

A *c*-confidence interval for the population mean μ

- $\bar{x} - E < \mu < \bar{x} + E$ where $E = z_c \dfrac{\sigma}{\sqrt{n}}$

- The probability that the confidence interval contains μ is c, assuming that the estimation process is repeated a large number of times.

Constructing Confidence Intervals for μ

Finding a Confidence Interval for a Population Mean (σ Known)

In Words	*In Symbols*
1. Verify that σ known, sample is random, and either the population is normally distributed or $n \geq 30$.	
2. Find the sample statistics n and $\bar{x}$.	$\bar{x} = \frac{\Sigma x}{n}$

Constructing Confidence Intervals for μ

In Words	*In Symbols*
3. Find the critical value z_c that corresponds to the given level of confidence.	Use Table 4, Appendix B.
4. Find the margin of error E.	$E = z_c \frac{\sigma}{\sqrt{n}}$
5. Find the left and right endpoints and form the confidence interval.	Left endpoint: $\bar{x} - E$ Right endpoint: $\bar{x} + E$ Interval: $\bar{x} - E < \mu < \bar{x} + E$

Example: Constructing a Confidence Interval

Construct a 95% confidence interval for the mean number of friends for all users of the website.

Solution: Recall $\bar{x} = 130.8$ and $E = 16.4$

Left Endpoint:

$$\bar{x} - E$$
$$= 130.8 - 16.4$$
$$= 114.4$$

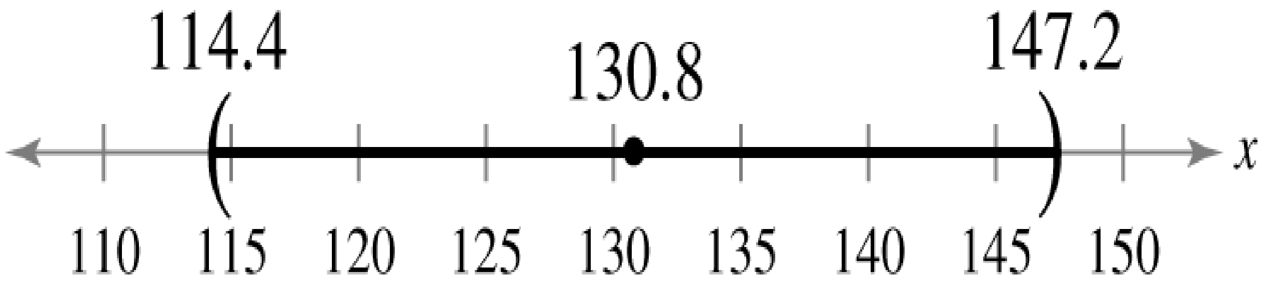
Right Endpoint:

$$\bar{x} + E$$
$$= 130.8 + 16.4$$
$$= 147.2$$

$$\mathbf{114.4 < \mu < 147.2}$$

Solution: Constructing a Confidence Interval

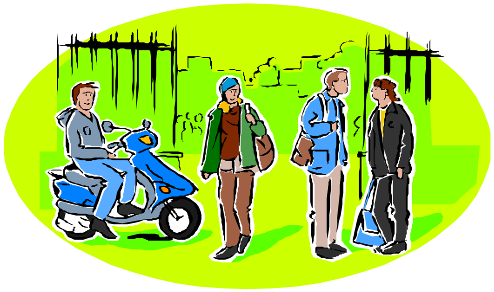
$114.4 < \mu < 147.2$



With 95% confidence, you can say that the population mean number of friends is between 114.4 and 147.2.

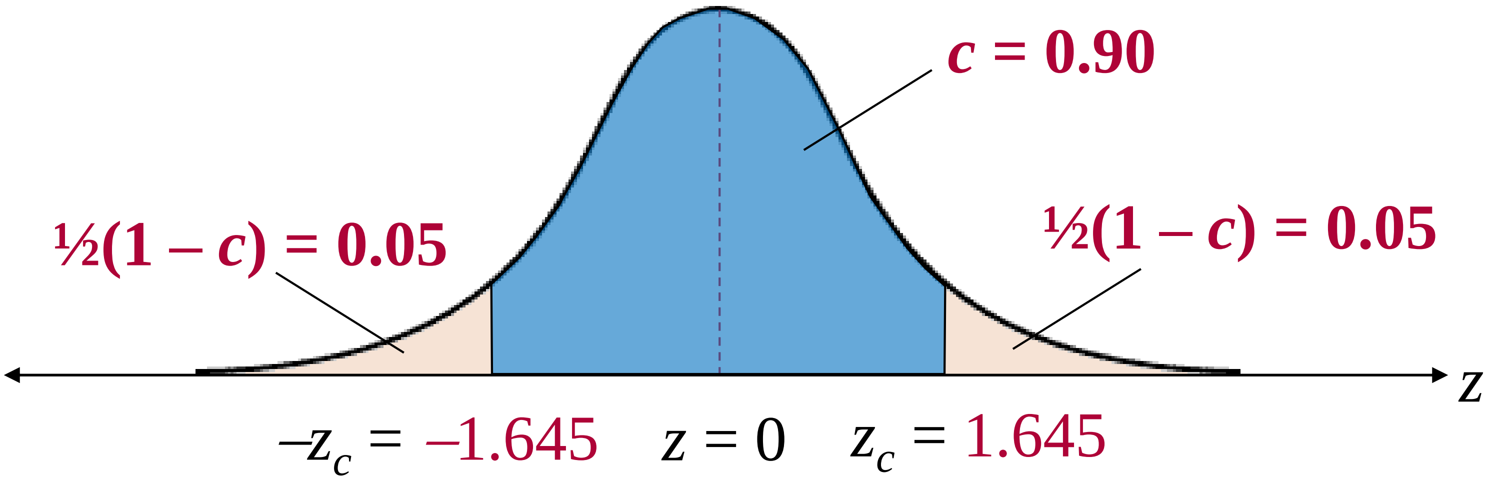
Example: Constructing a Confidence Interval

A college admissions director wishes to estimate the mean age of all students currently enrolled. In a random sample of 20 students, the mean age is found to be 22.9 years. From past studies, the standard deviation is known to be 1.5 years, and the population is normally distributed. Construct a 90% confidence interval of the population mean age.



Solution: Constructing a Confidence Interval

- First find the critical values



$$z_c = 1.645$$

Solution: Constructing a Confidence Interval

- Margin of error:

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{1.5}{\sqrt{20}} \approx 0.6$$

- Confidence interval:

Left Endpoint:

$$\bar{x} - E$$
$$\approx 22.9 - 0.6$$
$$= 22.3$$

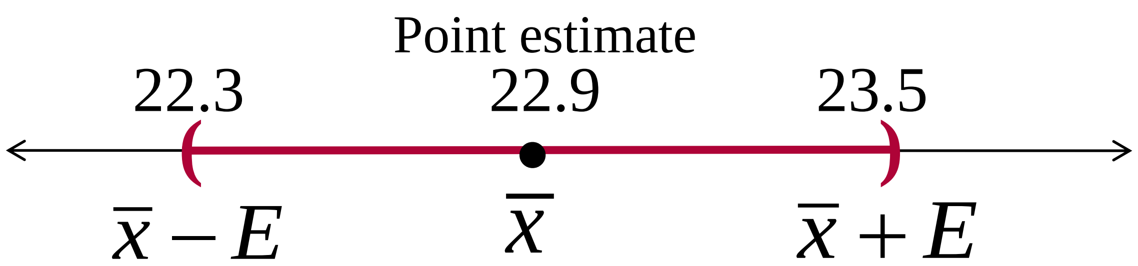
Right Endpoint:

$$\bar{x} + E$$
$$\approx 22.9 + 0.6$$
$$= 23.5$$

$$\mathbf{22.3 < \mu < 23.5}$$

Solution: Constructing a Confidence Interval

$22.3 < \mu < 23.5$



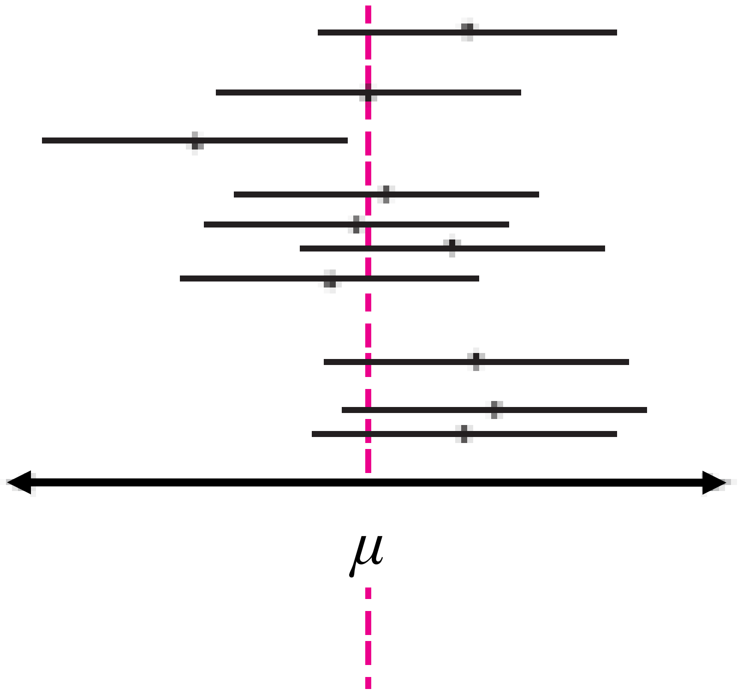
With 90% confidence, you can say that the mean age of all the students is between 22.3 and 23.5 years.

Interpreting the Results

- μ is a fixed number. It is either in the confidence interval or not.
- **Incorrect:** "There is a 90% probability that the actual mean is in the interval (22.3, 23.5)."
- **Correct:** "If a large number of samples is collected and a confidence interval is created for each sample, approximately 90% of these intervals will contain μ.

Interpreting the Results

The horizontal segments represent 90% confidence intervals for different samples of the same size. In the long run, 9 of every 10 such intervals will contain μ.



Sample Size

- Given a c-confidence level and a margin of error E, the minimum sample size n needed to estimate the population mean μ is

$$n = \left(\frac{z_c \sigma}{E} \right)^2$$

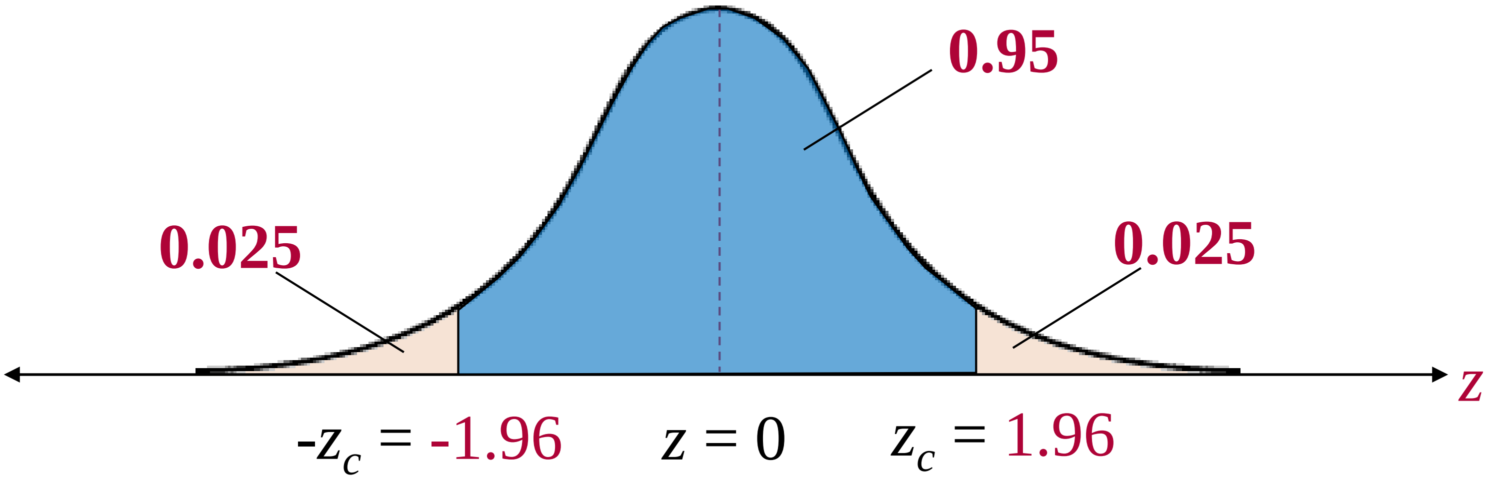
- If σ is unknown, you can estimate it using s provided you have a preliminary sample with at least 30 members.

Example: Determining a Minimum Sample Size

You want to estimate the mean number of friends for all users of the website. How many users must be included in the sample if you want to be 95% confident that the sample mean is within seven friends of the population mean? Assume the sample standard deviation is about 53.0.

Solution: Determining a Minimum Sample Size

- First find the critical values



$$z_c = 1.96$$

Solution: Determining a Minimum Sample Size

$z_c = 1.96 \qquad \sigma \approx s = 53.0 \qquad E = 7$

$$n = \left(\frac{z_c \sigma}{E}\right)^2 \approx \left(\frac{1.96 \cdot 53.0}{7}\right)^2 \approx 220.23$$

When necessary, **round up** to obtain a whole number.

You should include **at least 221** users in your sample.

The *t*-Distribution

- When the population standard deviation is unknown, the sample size is less than 30, and the random variable x is approximately normally distributed, it follows a ***t*-distribution**.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

- Critical values of t are denoted by t_c.

Properties of the *t*-Distribution

1. The mean, median, and mode of the *t*-distribution are equal to zero.
2. The *t*-distribution is bell shaped and symmetric about the mean.
3. The total area under a *t*-curve is 1 or 100%.
4. The tails in the *t*-distribution are “thicker” than those in the standard normal distribution.
5. The standard deviation of the *t*-distribution varies with the sample size, but it is greater than 1.

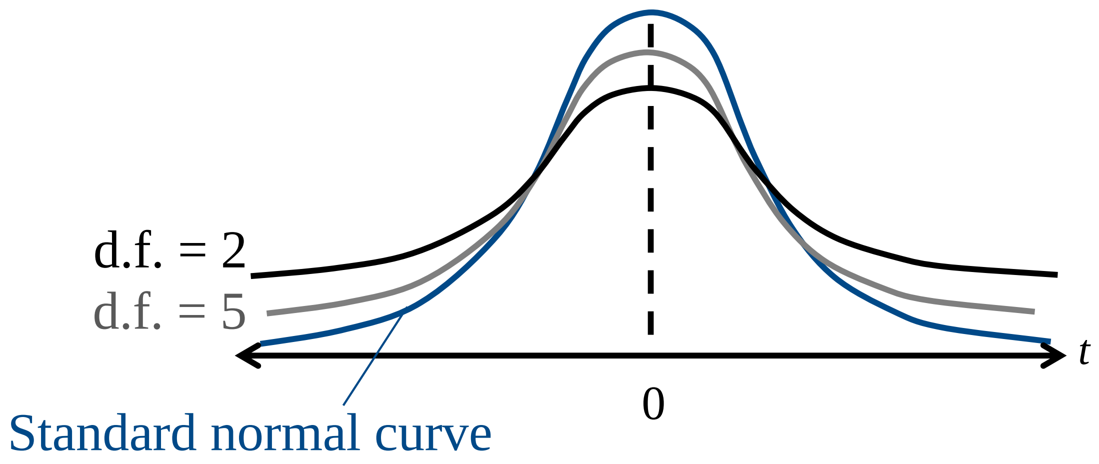
Properties of the *t*-Distribution

6. The *t*-distribution is a family of curves, each determined by a parameter called the degrees of freedom. The **degrees of freedom** are the number of free choices left after a sample statistic such as is calculated. When you use a *t*-distribution to estimate a population mean, the degrees of freedom are equal to one less than the sample size.

- d.f. = $n - 1$ Degrees of freedom

Properties of the *t*-Distribution

7. As the degrees of freedom increase, the *t*-distribution approaches the normal distribution. After 30 d.f., the *t*-distribution is very close to the standard normal *z*-distribution.



Example: Finding Critical Values of *t*

Find the critical value t_c for a 95% confidence when the sample size is 15.

Solution: d.f. = n – 1 = 15 – 1 = 14

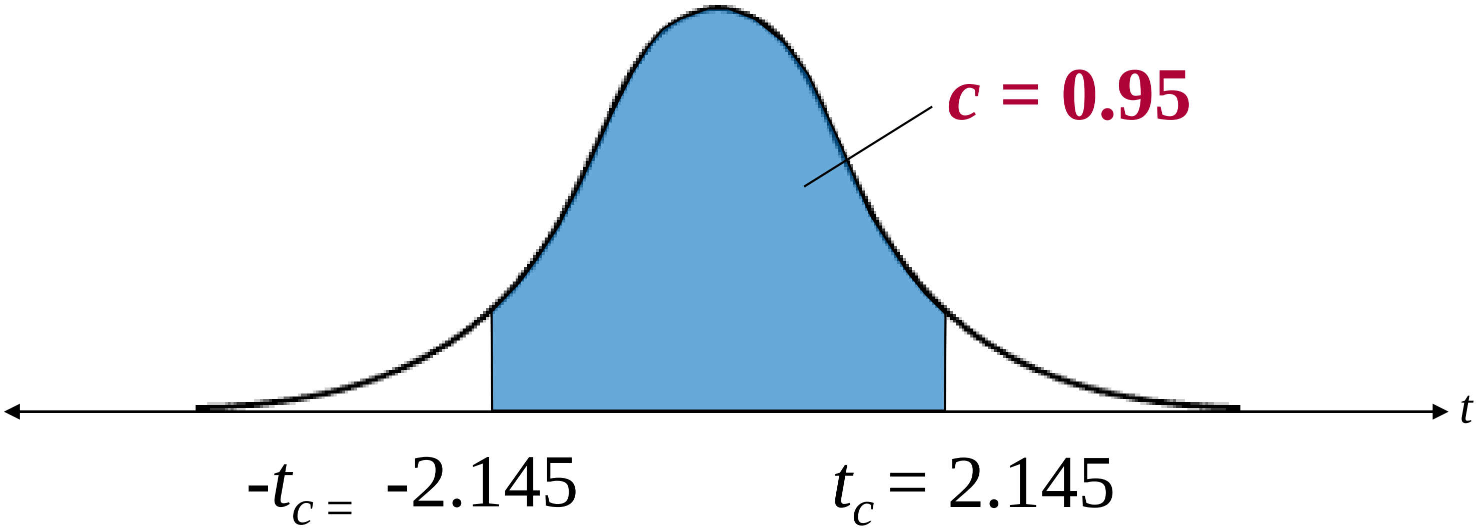
Table 5: *t*-Distribution

	Level of confidence, *c*	0.50	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
12		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
28		.683	1.313	1.701	2.048	2.467	2.763
29		.683	1.311	1.699	2.045	2.462	2.756
∞		.674	1.282	1.645	1.960	2.326	2.576

$t_c = 2.145$

Solution: Critical Values of *t*

95% of the area under the *t*-distribution curve with 14 degrees of freedom lies between $t = \pm 2.145$.



Confidence Intervals for the Population Mean

A *c*-confidence interval for the population mean μ

- $\bar{x} - E < \mu < \bar{x} + E \quad \text{where } E = t_c \dfrac{s}{\sqrt{n}}$

- The probability that the confidence interval contains μ is c.

Confidence Intervals and *t*-Distributions

In Words	*In Symbols*
1. Verify that σ is not known, the sample is random, and the population is normally distributed or $n \geq 30$.	
2. Identify the sample statistics n, $\bar{x}$, and s.	$\bar{x} = \frac{\Sigma x}{n}$ $\quad s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

Confidence Intervals and *t*-Distributions

In Words	*In Symbols*
3. Identify the degrees of freedom, the level of confidence c, and the critical value t_c.	d.f. = $n - 1$; Use Table 5.
4. Find the margin of error E.	$E = t_c \frac{s}{\sqrt{n}}$
5. Find the left and right endpoints and form the confidence interval.	Left endpoint: $\bar{x} - E$ Right endpoint: $\bar{x} + E$ Interval: $\bar{x} - E < \mu < \bar{x} + E$

Example: Constructing a Confidence Interval

You randomly select 16 coffee shops and measure the temperature of the coffee sold at each. The sample mean temperature is 162.0°F with a sample standard deviation of 10.0°F. Find the 95% confidence interval for the mean temperature. Assume the temperatures are approximately normally distributed.

Solution:

Use the *t*-distribution ($n < 30$, σ is unknown, temperatures are approximately distributed.)

Solution: Constructing a Confidence Interval

- $n = 16$, $\bar{x} = 162.0$ $s = 10.0$ $c = 0.95$
- $\text{df} = n - 1 = 16 - 1 = 15$
- Critical Value

Table 5: t-Distribution

d.f.	Level of confidence, c	0.50	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
12		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
28		.683	1.313	1.701	2.048	2.467	2.763
29		.683	1.311	1.699	2.045	2.462	2.756
∞		.674	1.282	1.645	1.960	2.326	2.576

$t_c = 2.131$

Solution: Constructing a Confidence Interval

- Margin of error:

$$E = t_c \frac{s}{\sqrt{n}} = 2.131 \cdot \frac{10}{\sqrt{16}} \approx 5.3$$

- Confidence interval:

Left Endpoint:

$$\bar{x} - E$$
$$\approx 162 - 5.3$$
$$= 156.7$$

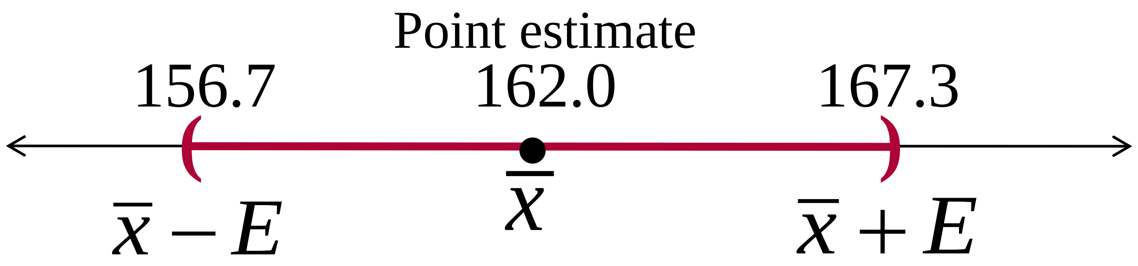
Right Endpoint:

$$\bar{x} + E$$
$$\approx 162 + 5.3$$
$$= 167.3$$

$$\mathbf{156.7 < \mu < 167.3}$$

Solution: Constructing a Confidence Interval

- **156.7 < *μ* < 167.3**



With 95% confidence, you can say that the mean temperature of coffee sold is between 156.7ºF and 167.3ºF.

Normal or *t*-Distribution?

Is σ known?

Yes →

If either the population is normally distributed or $n \geq 30$, then use the standard normal distribution with

$$E = z_c \frac{\sigma}{\sqrt{n}}.$$

Section 6.1

No →

If either the population is normally distributed or $n \geq 30$, then use the t-distribution with

$$E = t_c \frac{s}{\sqrt{n}}$$

Section 6.2

and $n - 1$ degrees of freedom.

Example: Normal or *t*-Distribution?

You randomly select 25 newly constructed houses. The sample mean construction cost is $181,000 and the population standard deviation is $28,000. Assuming construction costs are normally distributed, should you use the normal distribution, the *t*-distribution, or neither to construct a 95% confidence interval for the population mean construction cost?

Solution:

Use the normal distribution (the population is normally distributed and the population standard deviation is known)

Point Estimate for Population *p*

Population Proportion

- The probability of **success** in a single trial of a binomial experiment.
- Denoted by ***p***

Point Estimate for *p*

- The population proportion of successes in a sample.
- Denoted by
  - $$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{number in sample}}$$
  - read as "*p* hat"

Point Estimate for Population *p*

Estimate Population Parameter…	with Sample Statistic
Proportion: p	$\hat{p}$

Point Estimate for *q*, the proportion of failures

- Denoted by $\hat{q} = 1 - \hat{p}$
- Read as "*q* hat"

Example: Point Estimate for *p*

In a survey of 1000 U.S. adults, 662 said that it is acceptable to check personal e-mail while at work. Find a point estimate for the population proportion of U.S. adults who say it is acceptable to check personal e-mail while at work. *(Adapted from Liberty Mutual)*

Solution: $n = 1000$ and $x = 662$

$$\hat{p} = \frac{x}{n} = \frac{662}{1000} \approx 0.662 \approx 66.2\%$$

Confidence Intervals for *p*

A *c*-confidence interval for the population proportion *p*

- $\hat{p} - E < p < \hat{p} + E \quad \text{where } E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$

- The probability that the confidence interval contains *p* is *c*, assuming that the estimation process is repeated a large number of times.

Constructing Confidence Intervals for *p*

In Words	*In Symbols*
1. Identify the sample statistics n and x.	
2. Find the point estimate $\hat{p}$.	$\hat{p} = \dfrac{x}{n}$
3. Verify that the sampling distribution of $\hat{p}$ can be approximated by the normal distribution.	$n\hat{p} \geq 5, \quad n\hat{q} \geq 5$
4. Find the critical value z_c that corresponds to the given level of confidence c.	Use Table 4

Constructing Confidence Intervals for *p*

In Words	*In Symbols*
5. Find the margin of error E.	$E = z_c\sqrt{\dfrac{\hat{p}\hat{q}}{n}}$
6. Find the left and right endpoints and form the confidence interval.	Left endpoint: $\hat{p} - E$ Right endpoint: $\hat{p} + E$ Interval: $\hat{p} - E < p < \hat{p} + E$

Example: Confidence Interval for *p*

In a survey of 1000 U.S. adults, 662 said that it is acceptable to check personal e-mail while at work. Construct a 95% confidence interval for the population proportion of adults in the U.S. adults who say that it is acceptable to check personal e-mail while at work.

Solution: Recall $\hat{p} \approx 0.662$

$$\hat{q} = 1 - \hat{p} = 1 - 0.662 = 0.338$$

Solution: Confidence Interval for *p*

- Verify the sampling distribution of $\hat{p}$ can be approximated by the normal distribution

$$n\hat{p} = 1000 \cdot 0.662 \approx 662 > 5$$

$$n\hat{q} = 1000 \cdot 0.3388 \approx 338 > 5$$

- Margin of error:

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.662)\cdot(0.338)}{1000}} \approx 0.029$$

Solution: Confidence Interval for *p*

- Confidence interval:

Left Endpoint:

$$\hat{p} - E$$
$$= 0.662 - 0.029$$
$$= 0.633$$

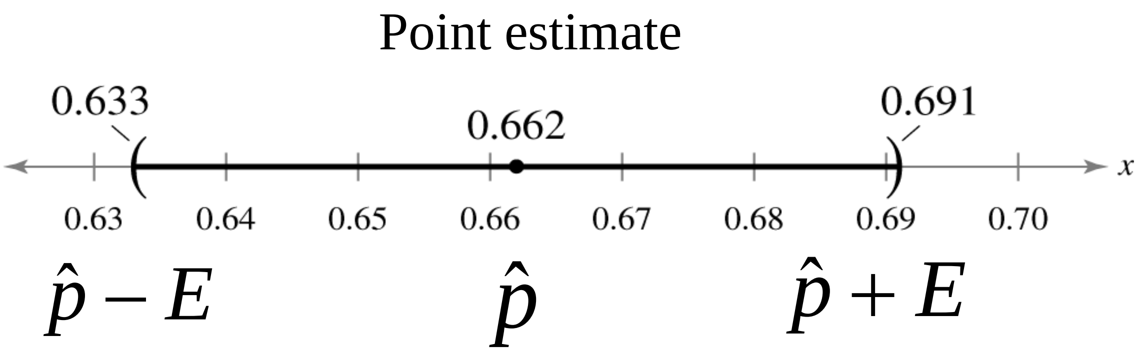
Right Endpoint:

$$\hat{p} + E$$
$$= 0.662 + 0.029$$
$$= 0.691$$

$$\mathbf{0.633 < \boldsymbol{p} < 0.691}$$

Solution: Confidence Interval for *p*

- **$0.633 < p < 0.691$**



With 95% confidence, you can say that the population proportion of U.S. adults who say that it is acceptable to check personal e-mail while at work is between 63.3% and 69.1%.

Determining a Minimum Sample Size

- Given a c-confidence level and a margin of error E, the minimum sample size n needed to estimate p is

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$$

- This formula assumes you have an estimate for $\hat{p}$ and $\hat{q}$.
- If not, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

Example: Sample Size

You are running a political campaign and wish to estimate, with 95% confidence, the proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the true population. Find the minimum sample size needed if

1. no preliminary estimate is available.

Solution:

Because you do not have a preliminary estimate for $\hat{p}$ use $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

Solution: Determining a Minimum Sample Size

- $c = 0.95 \qquad z_c = 1.96 \qquad E = 0.03$

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \approx 1067.11$$

Round up to the nearest whole number.

With no preliminary estimate, the minimum sample size should be **at least 1068 voters**.

Example: Determining a Minimum Sample Size

You are running a political campaign and wish to estimate, with 95% confidence, the proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the true population. Find the minimum sample size needed if

2. a preliminary estimate gives $\hat{p} = 0.31$.

Solution:

Use the preliminary estimate $\hat{p} = 0.31$

$$\hat{q} = 1 - \hat{p} = 1 - 0.31 = 0.69$$

Solution: Determining a Minimum Sample Size

- $c = 0.95 \qquad z_c = 1.96 \qquad E = 0.03$

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.31)(0.69)\left(\frac{1.96}{0.03}\right)^2 \approx 913.02$$

Round up to the nearest whole number.

With a preliminary estimate of $\hat{p} = 0.31$, the minimum sample size should be **at least 914 voters**. Need a larger sample size if no preliminary estimate is available.

The Chi-Square Distribution

- The **point estimate for σ^2** is s^2
- The **point estimate for σ** is s
- s^2 is the most unbiased estimate for σ^2

Estimate Population Parameter…	with Sample Statistic
Variance: σ^2	s^2
Standard deviation: σ	s

The Chi-Square Distribution

- You can use the ***chi-square distribution*** to construct a confidence interval for the variance and standard deviation.
- If the random variable *x* has a normal distribution, then the distribution of

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

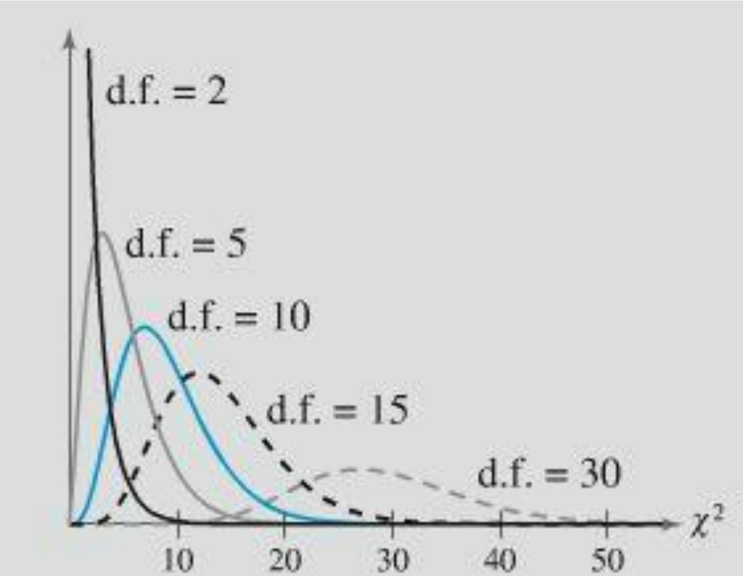
forms a **chi-square distribution** for samples of any size $n > 1$.

Properties of The Chi-Square Distribution

1. All chi-square values χ^2 are greater than or equal to zero.
2. The chi-square distribution is a family of curves, each determined by the degrees of freedom. To form a confidence interval for σ^2, use the χ^2-distribution with degrees of freedom equal to one less than the sample size.
   - d.f. = $n - 1$ Degrees of freedom
3. The area under each curve of the chi-square distribution equals one.

Properties of The Chi-Square Distribution

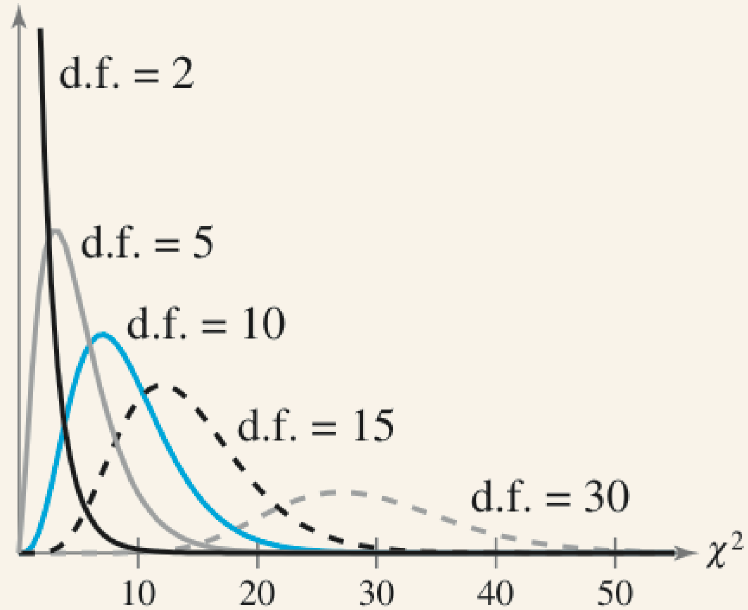
4. Chi-square distribution is positively skewed.
5. Chi-square distribution is different for each number of degrees of freedom. As degrees of freedom increase, the chi-square distribution approaches a normal distribution.



Chi-Square Distributions

Properties of The Chi-Square Distribution

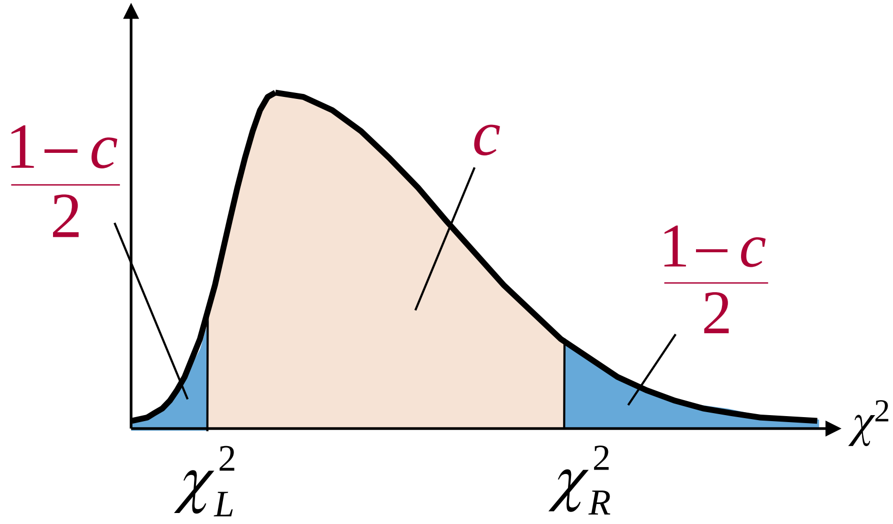
4. Chi-square distribution is positively skewed.
5. Chi-square distribution is different for each number of degrees of freedom. As degrees of freedom increase, the chi-square distribution approaches a normal distribution.



Chi-Square Distribution for Different Degrees of Freedom

Critical Values for χ^2

- There are two critical values for each level of confidence.
- The value χ^2_R represents the right-tail critical value
- The value χ^2_L represents the left-tail critical value.



The area between the left and right critical values is c.

Example: Finding Critical Values for χ^2

Find the critical values χ_R^2 and χ_L^2 for a 95% confidence interval when the sample size is 18.

Solution:

- d.f. = n – 1 = 18 – 1 = 17 d.f.
- Each area in the table represents the region under the chi-square curve to the *right of the critical value*.
- Area to the right of $\chi^2_R = \frac{1-c}{2} = \frac{1-0.95}{2} = 0.025$
- Area to the right of $\chi^2_L = \frac{1+c}{2} = \frac{1+0.95}{2} = 0.975$

Solution: Finding Critical Values for χ^2

Table 6: χ^2-Distribution

Degrees of freedom	α							
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170

χ^2_L χ^2_R

$\chi^2_L = 7.564$ $\chi^2_R = 30.191$

95% of the area under the curve lies between 7.564 and 30.191.

Confidence Intervals for σ^2 and σ

Confidence Interval for σ^2:

- $$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

Confidence Interval for σ:

- $$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

- The probability that the confidence intervals contain σ^2 or σ is c, assuming that the estimation process is repeated a large number of times.

Confidence Intervals for σ^2 and σ

In Words	*In Symbols*
1. Verify that the population has a normal distribution.	
2. Identify the sample statistic *n* and the degrees of freedom.	d.f. = $n - 1$
3. Find the point estimate s^2.	$s^2 = \frac{\sum(x - \bar{x})^2}{n-1}$
4. Find the critical value χ^2_R and χ^2_L that correspond to the given level of confidence *c*.	Use Table 6 in Appendix B

Confidence Intervals for σ^2 and σ

In Words	*In Symbols*
5. Find the left and right endpoints and form the confidence interval for the population variance.	$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$
6. Find the confidence interval for the population standard deviation by taking the square root of each endpoint.	$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$

Example: Constructing a Confidence Interval

You randomly select and weigh 30 samples of an allergy medicine. The sample standard deviation is 1.20 milligrams. Assuming the weights are normally distributed, construct 99% confidence intervals for the population variance and standard deviation.

Solution:

- d.f. = n – 1 = 30 – 1 = 29 d.f.

Solution: Constructing a Confidence Interval

- Area to the right of $\chi^2_R = \frac{1-c}{2} = \frac{1-0.99}{2} = 0.005$
- Area to the right of $\chi^2_L = \frac{1+c}{2} = \frac{1+0.99}{2} = 0.995$
- The critical values are
  $\chi^2_R = 52.336$ and $\chi^2_L = 13.121$

Solution: Constructing a Confidence Interval

Confidence Interval for σ^2:

Left endpoint: $$\frac{(n-1)s^2}{\chi_R^2} = \frac{(30-1)(1.20)^2}{52.336} \approx 0.80$$

Right endpoint: $$\frac{(n-1)s^2}{\chi_L^2} = \frac{(30-1)(1.20)^2}{13.121} \approx 3.18$$

$$\mathbf{0.80 < \sigma^2 < 3.18}$$

With 99% confidence you can say that the population variance is between 0.80 and 3.18 milligrams.

Solution: Constructing a Confidence Interval

Confidence Interval for σ:

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(30-1)(1.20)^2}{52.336}} < \sigma < \sqrt{\frac{(30-1)(1.20)^2}{13.121}}$$

$$\mathbf{0.89 < \sigma < 1.78}$$

With 99% confidence you can say that the population standard deviation is between 0.89 and1.78 milligrams.